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ABSTRACT

This study reports on the comparison of three methods of teaching general mathematics: (1) non-verbalized student discovery of principles and generalizations, (2) student-teacher development of principles and generalizations, and (3) teacher statement and application of principles and generalizations with illustrations in a problem set followed by student application. The bases for comparisons in each unit of instruction were achievement and retention. None of the three teaching methods was superior for male students on either achievement or retention. Female subjects in general achieved significantly better (.05) under the directed methods, but retention differences were non-significant. (Author/RS)

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A COMPARISON OF THREE METHODS OF TEACHING SELECTED MATHEMATICAL CONTENT IN EIGHTH AND NINTH GRADE GENERAL MATHEMATICS COURSES

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PREFACE

The research reported in this document was conducted as two separate dissertations at the University of Georgia by the authors. Dr. James Strickland completed requirements for a doctorate in Mathematics Edcuation in August, 1968, under the direction of Dr. Owen Scott. At the time of this writing, Dr. Strickland is on active duty with the United States Army. Dr. Fred Maynard completed requirements for his doctorate in Mathematics Education in August, 1969, under the direction of Dr. Len Pikaart. Currently, Dr. Maynard holds an appointment of Associate Professor at Augusta College, Augusta, Georgia.

In compiling this final report of the research grant, the authors have omitted several of the complete analyses which yielded no significant results in order to reduce the size of the report. Interested readers are encouraged to find these analyses, more examples of the teaching materials, and copies of the investigator constructed evaluation instruments in the dissertations of the authors which are available from University Microfilms in Ann Arbor, Michigan (the title of Dr. Strickland's dissertation is "A Comparison of Three Methods of Teaching Selected Content in General Mathematics"and that of Dr. Maynard is the same as the title of this report.)

Special thanks are due to the cooperating school administrators teachers and students. Also, several faculty members at the University of Georgia made significant contributions to the whole study. In particular, Dr. William D. McKillip was instrumental in formulating the problem and gave freely of his time to offer many useful suggestions.

Len Pikaart, Director
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SUMMARY

This study compared three methods of teaching selected mathematical content in eighth and ninth grade general mathematics courses. The three methods were labeled "Method D" (non-verbalized student discovery of principles and generalizations), "Method E" (student-teacher development of principles and generalizations) and "Method S" (teacher statement and application of principles and generalizations with illustrations in a problem set followed by student application of the principles and generalizations in a similar problem set). Using the content selected from Units 1, 2, 3, and 4 of Experiences in Mathematical Discovery (National Council of Teachers of Mathematics, 1966), three units (1, 2, 3) of instructional materials were written according to the definition of each of the three teaching methods.

The experimental phase of the study was conducted from November 29, 1967 to February 16, 1968. The sample consisted of approximately 400 students in 18 general mathematics classes taught by seven teachers in the two Georgia public schools. Each teacher was randomly assigned two of the three teaching methods which were, in turn, randomly assigned to that teacher's classes. Assignments were made so that a total of six classes constituted each treatment group. The teachers received training in their assigned methods twice before the beginning of Unit 1 and once during each unit of instruction. The teachers, classes, methods, and assignments were constant for each unit, only the subject matter changed. In fact, the subject matter of each unit was independent of the subject matter of the other units.

The bases for comparisons in each unit were achievement, as measured by an immediate posttest in the selected content, and retention, as measured by a delayed posttest which was administered approximately five weeks after the end of the unit. These tests were constructed by the investigators. In addition to unit pretests, the Otis Quick-Scoring Mental Ability Tests and the Arithmetic Concepts subtest of the Stanford Achievement Test were used to classify subjects according to mental ability and general mathematical achievement. A Student Rating Scale (SRS) and an Observer Rating Scale (ORS) were instruments by which students and adults rated teacher fidelity to his assigned methods. Dutton's Attitude Toward Mathematics scale was employed to measure changes in both teacher and student attitude over the entire project.

Analysis of variance and analysis of covariance were the statistical techniques employed in the study.

None of the three teaching methods was superior for male students on either achievement or retention. Female subjects, however, did not achieve or retain the selected material as well in the less-directed method (D) during the first unit of instruction. In subsequent units female subjects in general continued to achieve significantly better under the more-directed methods (E and S) but retention differences were non-significant.

This research indicates that the three methods defined in the study are equally effective for retention of the selected mathematics material if the methods are consistently used for a period of approximately one month.

CHAPTER I

ORIENTATION TO THE STUDY

In a recent paper, Romberg and DeVault (1967) identified form contents which influence mathematics curriculum: (1) mathematics, (2) learners, (3) teachers, and (4) instruction. The concern of the present study fell within the fourth component--instruction. In particular, the study concentrated on three specific instructional approaches which are differentiated primarily by the amount of teacher exposition or guidance toward the desired learning outcomes. The learners were eighth and ninth grade general mathematics students who were predominantly below-average achievers in mathematics; teachers were full-time classroom teachers in public schools; and the mathematics consisted of topics deemed suitable for general mathematics students by the National Council of Teachers of Mathematics.

Problem

This study compared three methods of teaching selected mathematical content in eighth- and ninth- grade general mathematics courses.

Brief descriptions of the teaching methods appear below (detailed descriptions are in the section "Teaching Methods," page 7):

Method D stresses non-verbalized student discovery of mathematical principles and generalizations with the teacher in a supervisory role.

Method E stresses teacher-student development of mathematical principles and generalizations through overt cooperative effort.

Method S stresses teacher statement and application of mathematical principles and generalization to a problem set, followed by student application of principles and generalizations to a similar problem set.

The mathematical content was selected from Units 1, 2, 3, and 4 of Experiences in Mathematical Discovery (National Council of Teachers of Mathematics, 1966). (See letters requesting and granting permission to use the content of Experiences in Mathematical Discovery in Appendix A, p. 78.) The selected content treats simple formulas, patterns, graphs, properties of operations with whole numbers, mathematical sentences, and geometry. For purposes of this study, the selected content was organized into three instructional units--Unit 1, Unit 2, and Unit 3.

Teaching methods were compared on the basis of student achievement and retention. Achievement was measured by immediate posttests administered immediately following each unit. Retention was measured by delayed posttests administered approximately five weeks following the completion of each unit.

To accomplish the purpose of the study, the experimenters used 18 general mathematics classes in two Georgia public schools for approximately eight weeks during the 1967-68 academic year.

Hypotheses

The study consisted of three independent units of mathematics and was treated as three individual studies with the same sample and instructors. The primary objective was the same for each unit--to determine the relative effectiveness of Methods, D, E, and S upon achievement and retention. The specific hypotheses may be stated in categories. (a) Those pertaining to units 1 and 3 were:

1. There are no significant differences among adjusted group achievement means when the subjects are classified according to each of the following main variables:

    (a) treatment (method)
    (b) teacher
    (c) sex of student
    (d) school
    (e) grade level
    (f) period of day

2. There are no significant differences among adjusted treatment group means on achievement for subjects within each of the following classifications:

    (g) eighth grade
    (h) ninth grade
    (i) female teachers
    (j) male teachers
    (k) female students
    (l) male students
    (m) School 1
    (n) School 2
    (o) eighth grade female students
    (p) eighth grade male students
    (q) ninth grade female students
    (r) ninth grade male students
    (s) female students of female teachers
    (t) male students of female teachers
    (u) female students of male teachers
    (v) male students of male teachers

3. There are no significant differences among adjusted group retention means when the subjects are classified according to each of variables 1 (a) through 1 (l).

4. There are no significant differences among adjusted treatment group retention means for subjects within each of classifications 2 (q) through 2(v).

5. There are no significant first-order interactions between any two of the variables treatment, sex of student, prior general mathematical achievement level, mental ability level, and prior achievement level in the selected content.

In the preceeding hypotheses, the phrase "adjusted... means" refers to means adjusted for mental ability, prior general mathematical achievement, and prior achievement in the selected content by the analysis of covariance. The null hypotheses formulated specifically for Unit 2 will now be stated.

For Unit 2, the following null hypotheses pertaining to the composite population of classes were formulated:

1. The three treatments (teaching methods) have no differential effect on achievement in the selected content.

2. The three treatments have no differential effect on retention in the selected content.

For Unit 2, the following null hypotheses pertaining to the population of ninth (eighth) grade students were formulated:

1. The three treatments have no differential effect on achievement in the selected content.
2. Ninth (eighth) grade general mathematics classes nested in treatments have no differential effect on achievement in the selected content.
3. Achievement of students who score at or above the pretest median does not differ from the achievement of students who score below the pretest median.
4. The three treatments have no differential effect on retention in the selected content.
5. Ninth (Eighth) grade general mathematics classes nested in treatments have no differential effect on retention in the selected content.
6. Retention of students who score at or above the pretest median differs from the retention of students who score below the pretest median.

For unit 2 the following hypotheses pertaining to the population of eighth grade students were formulated in addition to the preceeding hypotheses:

1. The two schools have no differential effect on achievement in the selected content.
2. The two schools have no differential effect on retention in the selected content.

For the entire study the following null hypotheses were formulated:

1. When students are classified according to treatment and grade level, there are no significant differences in change of attitudes toward mathematics, as measured by gains on Dutton's scale, among classification groups.
2. The selected first-order interactions between the variables treatment (method), teacher, sex of student, period of day, grade level, class, and school are not significant.

Educational Significance

This study is of particular significance to mathematics educators, teachers of eighth and ninth grade general mathematics courses, and researchers in mathematics education.

In working with prospective and in-service general mathematics teachers, mathematics educators can use the three methods (D,E, and S) as specific examples of less-directed and more-directed teaching procedures. Model lessons taken from this study's instructional materials can be used to demonstrate the use of each method. Prospective and in-service teachers can be shown that there is not just one method of teaching a given mathematical principle, but several methods are available. Mathematics educators can use the results of the study when discussing the relative effectiveness of the three methods as used in actual classroom settings.

Many eighth- and ninth-grade general mathematics teachers are seeking methods of teaching which can be used effectively in their classes. This study describes three such methods and demonstrates the relative effectiveness of Methods D, E, and S in general mathematics classes. The descriptions of the three methods can be used by teachers to prepare lesson plans in a variety of content areas. If the selected content is to be taught, a teacher may choose to use the instructional materials written for this study.

Methodology is an area of current interest to research workers in mathematics education. This is evidenced in Chapter II, "Review of the Related Literature." As an aid to these researchers, the present study offers a comprehensive review of theoretical proposals and empirical studies, a method of attacking a problem involving the comparison of several teaching approaches, and the results of comparing Methods D,E, and S on the basis of student achievement and retention. In Chapter V, recommendations for future research are offered for consideration and, hopefully, pursuit by researchers in mathematics education.

Teaching Methods

Method D is a teaching method characterized by non-verbalized student discovery of mathematical principles and generalizations with the teacher in a supervisory role. "Discovery" is used here in the sense that Bruner (1961) describes it; that is, as a rearrangement or transformation of evidence by the student so that he is enabled to go beyond the evidence to new insights. The student discovery in this method is referred to as "non-verbalized" because the student is not required to verbalize his new insights. Students in this method work independently on an individual set of instructional materials as they attempt to discover mathematical principles for themselves. Talking among students is not allowed. The teacher circulates among students and is allowed to tell students one at a time whether their answers to specific problems are right or wrong. The teacher is not allowed to state mathematical principles or generalizations intended for student discovery. No pressure is exerted on students to verbalize their findings. However, if a student asks a question about the truth or falsity of a student-formulated generalization, the teacher is allowed to tell the student whether the generalization is true or false. If the generalization is false, the teacher encourages the student to look for a counter-example to the false generalization. Students are frequently encouraged to think of themselves as discoverers of mathematics.

The major components of Method D may be summarized as follows:

1. Students work on their own in an attempt to discover mathematical principles and generalizations for themselves.

2. Students do not talk to each other about their work.

3. The teacher tells students one at a time whether their answers to problems in the "discussion" section of a lesson are right or wrong.

4. The teacher writes the answers to problems in the "exercises: section of a lesson on the chalkboard but does not show students how to solve the problems.

5. The teacher gives frequent encouragement to students to think of themselves as discoverers of mathematics.

6. When students check the truth or falsity of student-formulated generalizations with the teacher, the teacher states whether the generalizations are true or false and encourages students to look for their own counterexamples to false generalizations.

Method E is a teaching method characterized by teacher-student development of mathematical principles and generalizations through overt cooperative effort. An "overt cooperative effort" may be thought of as an open, verbal sharing of ideas between teacher and students as they work together to arrive at mathematical principles and generalizations. Students in this method are furnished with individual sets of instructional materials, but they are not required to work independently as in Method D. The teacher leads the class through a predetermined sequence of questions and problems. Class members are called upon to supply answers, and they may be assisted or challenged by other class members. The teacher is allowed to supply answers when the class is unable to do so. Students are encouraged to use the problems inductively to discover and verbalize the mathematical principles or generalizations involved. If a student thinks he has arrived at a generalization, he is encouraged to verbalize his generalization for the class. If the generalization is false, the teacher offers a counterexample to show why it is false. The teacher verbalizes the intended mathematical principle only if the class is unable to do so.

The major components of Method E may be summarized as follows:

1. Students and teacher work out mathematical principles and generalizations through overt cooperative effort.
2. Students share ideas with each other in an effort to discover mathematical principles and generalizations.
3. The teacher tells students whether their answers to problems in the "discussion" section of a lesson are right or wrong and encourages students to use these problems inductively to discover mathematical principles or generalizations.
4. The teacher writes the answers to problems in the "exercises" section of a lesson on the chalkboard and identifies the mathematical principle that can be used to solve each problem.
5. The teacher encourages students to think of themselves as discoverers of mathematics.
6. When students check the truth or falsity of generalizations with the teacher, the teacher states whether the generalizations are true or false and gives counterexamples to false generalizations.

Method S refers to a teaching method characterized by teacher statement and application of mathematical principles and generalizations to a problem set, followed by student application of principles and generalizations to a similar problem set. The initial setp in this method is the statement of a mathematical principle or generalization by the teacher. The teacher then applies the principle or generalization to a problem set, showing the class step-by-step solutions. Each student has his own set of instructional materials in which he can follow the demonstration. When the teacher completes the demonstration, students attempt to apply the mathematical principle or generalization to a similar problem set. Class members are allowed to share ideas with each other about how to apply the principle or generalization. If a student asks a question about a problem, the teacher tells the student whether his answer is right or wrong, reiterates the mathematical principle or generalization that can be used to solve the problem, and relates the problem to other problems of the same type worked previously by the teacher. No pressure is exerted on students to discover or verbalize principles and generalizations for themselves. The teacher is regarded as the primary source of authority in mathematics.

The major components of Method S may be summarized as follows:

1. The teacher states and applies mathematical principles and generalizations without giving students an opportunity to discover the principles and generalizations for themselves.
2. Students share ideas with each other about how to apply mathematical principles and generalizations stated by the teacher.
3. The teacher tells students whether their answers to problems in the "discussion" section of a lesson are right or wrong, reiterates mathematical principles or generalizations that can be used to solve the problems, and relates the problems to others of the same type worked previously by the teacher.
4. The teacher writes the answers to problems in the "exercises" section of a lesson on the chalkboard, identifies the mathematical principle or generalization that can be used to solve a particular problem, and directs students' attention to problems of the same type which they have already solved.
5. The teacher gives no encouragement to students to think of themselves as discoverers of mathematics.
6. When students check the truth or falsity of generalizations with the teacher, the teacher states whether the generalizations are true or false but makes no attempt to explain why.

A sample lesson has been taken from the student manual of each treatment group (Method D, Method E, and Method S) and is included in Appendix B, p. 81 . This sample lesson illustrates many of the characteristics both common and unique to the various methods.

CHAPTER II

REVIEW OF THE RELATED LITERATURE

The present study compared three methods of teaching selected mathematical content in eighth- and ninth-grade general mathematics courses. As originally conceived, the problem and teaching methods were only rough ideas lacking precise definitions. To define the problem and teaching methods more precisely, to determine what research has been completed on the problem or related problems, and to gain ideas for solving the problem, the literature was surveyed. In this chapter, the pertinent literature is reviewed in two sections: (1) theoretical proposals relevant to the study, and (2) empirical research relevant to the study.

In order to be included in the review, a theoretical proposal had to satisfy the following criteria: (1) deal with instructional approaches which concentrate on the amount of guidance or direction given the learner in his efforts to discover, understand, or apply mathematical principles, (2) be a possible source of hypotheses for empirical research, (3) have an authoritative source, and (4) appear in the published literature since 1950. The year 1950 was selected because of the increased emphasis on school mathematics and methods of teaching mathematics at approximately that date. Henderson (1963) compared many of the more important theoretical proposals published prior to 1950, and a summary of his findings commences the section on theoretical proposals.

An empirical study had to satisfy the following cirteria in order to be included in the review: (1) be actual research rather than personal opinion, (2) deal with methods of teaching mathematics or methods of teaching which were differentiated by the amount of guidance or direction given the learner, (3) have as one of its objectives the assessment or comparison of the achievement effects of various teaching methods, and (4) be completed in the last 25 years. A study reported by Hendrix (1947) approximately 25 years ago seemed to be an appropriate starting point for the following reasons: (1) the study by Hendrix introduced a new concept in teaching mathematics--the concept of "unverbalized awareness," (2) one of the methods used in the present study, Method D, is very similar to the "unverbalized awareness" method described by Hendrix, and (3) Hendrix' findings prompted subsequent research on methods of teaching mathematics.

Theoretical Proposals Relevant to the Study

One source of theoretical proposals is the set of reports made by various committees and conference groups. When Henderson (1963) compared the methodological recommendations of several committees[1] who met prior to 1950, he found a pervasive belief in

> ..."the efficacy of a methodological sequence in which the teacher starts the students working informally with concrete objects or ideas which they understand and then guides their activities and thoughts so that they discover relations, principles, and procedures, rather than stating these relations, etc., as the initial step in the sequence. (p. 1011)."

In 1959, the Secondary School Curriculum Committee of the National Council of Teachers of Mathematics expressed a similar point of view on instruction. The committee called for more emphasis on pupil discovery and reasoning at all levels of instruction. In regard to teaching methods which should be used with slow learners in mathematics, the Committee proposed:

1. Generalizations, in order to be understood by the class, must be preceded by many and varied concrete illustrations.
2. Frequent reviews in meaningful situations are necessary in order to maintain a reasonable level of skill and understanding.
3. Laboratory techniques and manipulative devices should be used freely (National Council of Teachers of Mathematics, 1959, p. 409).

During the summer of 1963, the Cambridge Conference on School Mathematics was held to review school mathematics and to establish goals for mathematical education. In its report, the Cambridge Conference (1963) set forth several pedagogical principles and techniques which it believed would be instrumental in reaching the goals. For example, the report stated, "The discovery approach, in which the student is asked to explore a situation in his own way, is invaluable in developing creative and independent thinking in the individual (p. 17)." The Conference advanced the notion that teachers should usually aid student discovery by introducing required ideas when they are not forthcoming from the class, by bringing attention to misleading statements in the way of discussion, and by summarizing results clearly as they come forward. This should be done, the Conference maintained, with a minimum of authority. The student should never hesitate to state the results of his efforts, and half-formed ideas should be used as stepping stones to true statements.

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[1] Henderson's comparison is based on the reports of the National Committee on Mathematical Requirements (1921), the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics (1940), the Committee on the Function of Mathematics in General Education (1940), and the Commission on Post-War Plans (1944, 1945).

Although the Cambridge Conference held discovery in high esteem it pointed out that the development of independent and creative habits of thought does not require the exclusive use of discovery methods. The attainment of a reasonable rate of advancement in the curriculum requires the use of discovery supplemented by a dialogue between teacher and class, and the direct teacher presentation of material. The Conference proposed the following balance between discovery and direct teaching procedures, with the reservation that further experimentation was needed before final decisions could be made:

> In the earliest grades the discovery approach, teacher aided, should dominate. By grade 7 most of the classroom time will be occupied by more direct teaching procedures. However, in these later grades, creative thinking and independence should be fostered extensively by the exercises in school time and homework. A transition should take place in the intermediate years (Cambridge Conference on School Mathematics, 1963, p. 17).

In 1964, the U. S. Office of Education and the National Council of Teachers of Mathematics jointly sponsored a conference whose purpose was to consider the mathematical content and instructional needs of low achievers in mathematics. In its report, the Conference on the Low Achiever in Mathematics (Woodby, 1965) recommended a discovery-laboratory approach in which the low achiever experiments with concrete objects, records and organizes data, and then searches for mathematical structure on a more abstract level. The report stressed the importance of looking at the same mathematical concept in different ways, and suggested that the teaching technique to be used depends on the concept to be learned, the students involved, and the particular situation at the moment.

Another source of theoretical proposals is the individual theorist who is regarded as an authority. Bruner (1961), an advocate of discovery approaches, pointed out the following benefits of learning by discovery: (1) increase in intellectual potency, (2) a shift from extrinsic to intrinsic rewards, (3) learning the heuristics of discovery, and (4) the aid to memory processing. Increase in intellectual potency refers to an increase in the ability of the individual to assemble material sensibly. The intrinsic gratification of having found out something for one's self is a benefit offered by discovery learning. The heuristics of discovery are the attitudes and activities that go with inquiry and research. Learning by discovery aids memory processing, Bruner claimed, by making material more readily accessible in the memory.

Beberman (1962), in his description of the basic tenets of the University of Illinois Committee on School Mathematics (UICSM), set forth the notion that "the student must understand his mathematics (p.4.)." The student will come to understand mathematics, Beberman stated, when his textbook and teacher use unambiguous language and when he is enabled to discover generalizations by himself.

Although "discovery" is a basic principle in the UICSM program, Beberman pointed out that it is unnecessary to require a student to verbalize his discovery to determine whether he is aware of a rule. Compelling the student to make an immediate verbalization of what he discovers is ineffective, because the student may not have the linguistic capacity to do so. The technique of delaying the verbalization of important discoveries is characteristic of the UICSM program, and differentiates the UICSM discovery method from other methods which are called "discovery methods" but which involve the immediate verbalization of discoveries.

It should not be inferred that the UICSM program believes verbalization is unimportant in the learning of mathematics. Beberman explained, "Verbalization is necessary, for example, in the many cases in which a student believes that he has discovered a generalization, and wants to show that it is a theorem (p. 28)." It is a "premature verbalization" to which Beberman and the UICSM object.

The views of Hendrix (1961) on "delaying the verbalization of important discoveries" are similar to those of Beberman. Hendrix makes a distinction between the discovery process itself and the process of composing sentences which express the discoveries. According to Hendrix, the learner experiences a "nonverbal awareness" stage in the process of discovering a generalization. The nonverbal awareness stage usually occurs during the learner's search for a short cut that can be used to work a set of problems. The advent of awareness is usually accompanied by a behavioral change in the learner such as a sudden start or a flush of excitement. Insisting on a spontaneous verbalization of what has been discovered belittles the accomplishment of the actual discovery and can cause unnecessary frustration. Any attempts at verbalizing the discovery are better postponed to a later lesson, at which time the linguistic formulation of things already "known" can be undertaken as an end in itself. Hendrix concluded by stating, "It is recognition of the nonverbal awareness stage in inductive learning that converts the classroom experience into that of actual discovery, the kind of thing that promotes a taste for and a delight in research (p. 298)."

Lowry (1967) stated that there is strong research support for two major outcomes usually attributed to learning by discovery: better retention and greater potential for transfer. However, according to Lowry, discovery-type teaching does not appear to be as efficient as certain kinds of expository-type teaching in attaining immediate goals or in the rate at which content is encountered.

Fehr (1967), in his interpretation of research findings for classroom teachers, reported that for some students some methods of teaching are more effective than others. To illustrate his point, he stated that students will learn algebra whether it is taught by a verbalized, abstract, deductive method or by an experiential, nonverbalized, concrete, inductive method. But for students with an IQ of 117 or higher, there is a significant advantage in the inductive, experiential approach. Those students who study under this method, Fehr asserted, develop far greater skill in operation, greater understanding of concepts, and have longer retention of the knowledge learned than students of the same mental ability taught under the traditional "tell and do" method.

Some theorists have offered proposals concerning the advantages of giving guidance in discovery learning. One such theorist is Gagne (1966) who claimed that guided discovery has the advantages over unguided discovery of reducing the necessity for search and eliminating the most extreme wrong hypotheses.

Another such theorist is Craig (1953), who expressed the belief that adults of all ability levels benefit from clues that help them discover the bases for making correct responses to situations. The more guidance of this type they receive, the more errors they avoid while learning, and the more learning they transfer to similar situations. Craig's proposals are not to be construed as meaning that correct responses should be mechanically specified in advance. He is advocating the liberal use of guidance of learner activity as an aid in promoting efficient discovery.

Cronbach (1965) suggested that the learner's individual characteristics and the type of material to be learned are important factors in trying to determine the cognitive significance of discovery. Cronbach alleged that the greater the learner's maturity, relevant concrete experience, and command of symbolic systems, the less he would profit from the experience of discovery. Knowledge that can be verified experimentally or by its internal consistency would, in Cronbach's opinion, be more appropriate for discovery than knowledge that is conventional or factual-descriptive.

In another article, Cronbach (1966) expressed the belief that inductive teaching is rarely superior to other meaningful teaching for putting across single generalizations. The idea of an interaction between pupil characteristics and the discovery variable was again set forth. Believing that the interaction may have more to do with personality than ability, Cronbach stated:

> I am tempted by the notion that pupils who are negativistic may blossom under discovery training, whereas pupils who are anxiously dependent may be paralyzed by demands for self-reliance (Cronbach, 1966, p. 90).

The beliefs of Ausubel (1961) are in contrast to several of those reviewed above, particularly in regard to the merits of learning by discovery. Ausubel expressed doubt that "learning the heuristics of discovery" in a specific discipline has as much transferability across disciplinary lines as Bruner (1961) claimed. Placing more value on verbalization as an aid to transfer than Beberman (1962) and Hendrix (1961), Ausubel stated,

> ...Verbalization does more than attach a concenient symbolic handle to an idea; it is, rather, part of the very process of thought itself and makes possible a qualitatively higher, more precise, more general, and more transferable type of understanding (Ausubel,, 1961, p. 55).

Because of its serious time-cost disadvantage, Ausubel proposed that learning by discovery is not a feasible method of teaching subject matter content, except in two cases: (1) when the learner is in the concrete stage of logical operations and is dependent both on concrete empirical props and on a preliminary phase of intuitive, subverbal insight for the learning of complex abstractions, and (2) when the learner is an older individual trying to learn a difficult new discipline in which he is as yet very unsophisticated.

Kinsella (1965) built a strong case for teaching procedures which involve a mixture of student discovery and teacher exposition. He pointed out that contemporary school mathematics programs have the student's understanding as their primary objective, just as did the better teachers in older programs. Exclusive use of discovery techniques will not guarantee the attainment of understanding. Nor will "show-and-do methods," which emphasize "how" to perform mathematical operations. The "mix" advocated by Kinsella was expressed as follows:

> Teaching for understanding requires not only the acquisition of meanings and skills but also the provision of abundant and varied experience in relating and organizing the understandings already achieved. An emphasis on explanation, reasoning, and problem-solving is necessary, too, for the development of new understandings (Kinsella, 1965, p.82).

Rosskopf (1953) proposed a program of mathematics teaching which he maintained would develop the largest possible transfer of training. During the initial, or "understanding," stage, the teacher should be satistifed with students being able to solve tasks that require use of a concept. Neither students nor teacher should attempt to verbalize the concept at this stage, but this does not imply that verbalization by a particular student should be discouraged.

Following the "understanding" stage, students should be furnished enough practice so that they will have an opportunity to reorganize or reconstruct experiences in terms of the concept involved. If the concept is one that is a routine part of larger problems, the practice should be of the stimulus-response type. In such a case, drill has a definite place in a program of mathematics teaching. For those students who progress to higher levels of mathematics study. Rosskopf recommended that they learn to verbalize principles that are appropriate to their level of progress.

As far as general mathematics courses are concerned, Rosskopf expressed the belief that teaching for understanding and for formation of concepts should be paramount. The means of instruction in such courses should be discovery and exploration through many examples that use the same concept, followed by applications of the non-verbalized concepts to new problems.

The theories discussed above represent the diverse opinions found in the literature. An important function of theories is the influence they have on the empirical research attempted. The next section is devoted to a review of empirical studies.

Empirical Research Relevant to the Study

One of the earlier studies which is related to the present study was done by Hendrix (1947). The problem studied was, "To what extent, if any, does the way in which one learns a generalization affect the probability of his recognizing a chance to use it (p.197.)"

In the first observation of the experiment, three groups of college students were formed and the same mathematical generalization was taught to each group by a different method. The procedure used to constitute the three treatment groups, such as matching or random assignment, was not reported. One group discovered the generalization independently and left it unverbalized, another group discovered and verbalized the generalization, and the third group had the generalization stated and illustrated for them.

About two weeks later, the groups were given a test containing several items which could be readily answered by anyone who recognized a chance to use the generalization but which could also be answered, though more laboriously, by counting or adding. The "unverbalized awareness" group (the first group mentioned above) scored significantly higher at the .12 level. At this level the null hypothesis would not ordinarily be rejected, but a possible real difference is suggested. The lowest transfer effects came from the group who had the generalization stated and illustrated for them. The group who discovered and verbalized the generalization ranked between the other two on the transfer test.

Several months later Hendrix repeated the experiment twice, once with eleventh- and twelfth-grade boys and once with college girls. In each of these replications, the order of effectiveness of the methods was the same as in the initial experiment. The results suggested the following hypotheses to Hendrix:

1. For generation of transfer power, the unverbalized awareness method of learning a generalization is better than a method in which an authoritative statement of the generalization comes first.
2. Verbalizing a generalization immediately after discovery does not increase transfer power.
3. Verbalizing a generalization immediately after discovery may actually decrease transfer power (Hendrix, 1947,p. 198).

Hendrix concluded from her study that symbolic formulation is not the key to transfer. The key to transfer is a "subverbal, internal process, something which must happen to the organism before it has any new knowledge to verbalize (Hendrix, 1947, p. 200)."

In 1956, Sobel reported a study whose purpose was "to discover whether or not there is any relationship between the learning of certain algebraic concepts and their method of presentation (p. 425)." Two methods of presenting elementary algebraic concepts were compared. The "experimental method" was a concrete, nonverbalized, inductive method with students guided through experiences involving applications to discover and verbalize concepts. The "control method" was an abstract, verbalized, deductive method with concepts defined and presented by the teacher, followed by practice exercises.

Sobel formulated the following research hypotheses:

1. The experimental method of teaching will produce results which are significantly superior to the control method on a test which is designed to evaluate certain algebraic concepts.

2. This superiority will also be evidenced on a test which evaluates certain fundamental algebraic skills.

3. Both the concepts and skills will be retained better by the students taught through the experimental procedure (Sobel, 1956, p. 426).

Fourteen intact ninth grade algebra classes whose teachers indicated a willingness to take part in the study constituted the sample. Seven of the classes were assigned to each method, although it was not indicated whether the assignment was random. Each class was taught by a different teacher with the one exception in which a teacher taught two experimental classes. An attempt was made to assure reasonable equality between the two groups of teachers insofar as ability and experience were concerned. The experimental teachers were given a special manual to guide their development of the concepts and skills, and the control teachers used the development found in their textbooks.

After four weeks of instruction, an immediate posttest was given to each of the participating classes. An equivalent form of the immediate posttest was administered as a delayed posttest three months later. The data analysis on each posttest revealed that for high IQ students (IQ above 110) the experimental method was superior to the control method for attainment and retention of concepts and skills. The difference was significant at the .05 level. For average IQ students, no significant difference was found between the two methods. Although Sobel's methods and criterion measures differed from those of Hendrix (1947), it is worth noting that in both studies students who learned by less-directed methods performed at least as well or better on delayed posttests.

Nichols (1956) compared two approaches to the teaching of selected topics in plane geometry to high school freshmen. The purpose of the study was "to assess the effectiveness of learning certain geometric topics as related to the method by which they are taught (p. 2107)." The "dependence" approach, which Nichols classified as "deductive," stressed teacher statement of assumptions, theorems, definitions, and principles for students. The "structured search" approach, considered by Nichols to be "inductive," emphasized student discovery of relationships through a series of concrete experiences with drawings of geometric figures and through mensuration.

Two groups of 21 students each were formed so that the subjects in each group were matched on the criterion test score, IQ, sex, and age. The procedure used to assign the approaches to the groups was not reported. Each group was taught by three teachers, but the length of the instructional period was not indicated. Method effectiveness was assessed in terms of amount of growth in knowledge of vocabulary, critical thinking ability, ability to solve problems, and fundamental skills, as measured by the criterion test. On the basis of the obtained data, Nichols concluded that the structured search approach and dependence approach were equally effective in terms of the cirterion test in teaching plane geometry to high school freshmen of average and superior intelligence.

In a recent study, Neuhouser (1965) compared three methods of teaching a unit on exponents to eighth grade students. The methods were described as follows:

> Method A was essentially the statement of a rule, a rationale for the rule, illustrations, and examples for the student to work. Method B and C were discovery methods. Method B contained no verbalization of rules, while in Method C the subject was helped to make a statement of the rules after discovery (Neuhouser, 1965, p. 5027).

Programmed instruction was used to carry out all three methods. Approximately 40 students were randomly assigned to each treatment group. A pretest revealed that the subjects had no pre-experimental knowledge of the rules for exponents.

At the beginning of Neuhouser's experiment, the treatment groups were compared on the bases of IQ scores, scores on mathematics and reading achievement tests, and first semester mathematics grades. No significant differences were found among the groups on these measures. Upon completion of the unit, the three methods were compared using four different posttests. The methods were also compared on the amount of time taken by each subject to complete the unit. The principal conclusion was that "students taught by a nonverbal directed discovery method probably take no longer to learn, have at least as much manipulative ability, more understanding, more ability to transfer, and much more retention than students taught by a nondiscovery method (p. 5027)."

Howitz (1966) compared a guided discovery method of teaching new course content with a conventional method of teaching conventional content in ninth grade general mathematics. The sample consisted of 12 ninth grade general mathematics classes in four schools. The treatments were randomly assigned to classes, so that six classes were taught by each method. Six teachers took part in the study, each teaching both an experimental class and a control class.

The experimental treatment was characterized by a guided discovery approach in which students were led through a series of planned questions to discover mathematical generalizations. The control treatment was characterized by expository methods and involved no discovery. A contemporary text, Experiences in

Mathematical Discovery, was used in experimental classes, while a more conventional text, Refresher Arithmetic, was used in control classes.

The study ran the entire academic year 1963-64. When mathematical achievement was measured by the Sequential Tests of Educational Progress, no significant difference was found between treatment groups. However, when achievement was measured by a test based on the content of Experiences in Mathematical Discovery, the experimental group scored significantly higher.

In 1966 Price reported a study with the following purposes:

1. To define and categorize various aspects of discovery.

2. To prepare sample lessons which make use of the above defined dimensions of discovery.

3. To conduct an experiment using the above materials to determine the effect of discovery methods on the achievement and critical thinking abilities of students so taught (Price, 1966, p. 5304).

In order to carry out the experimental phase of Price's study, three classes in tenth grade general mathematics were randomly selected from a general mathematics population in a large city high school. One class was taught the conventional course of study through a textbook-lecture method. A second class was taught material similar to that used in the first class, but specially prepared discovery lessons were used. The third class used the same materials as the second class together with specially prepared transfer lessons. The transfer lessons were used to promote certain selected aspects of critical thinking. All three classes were given pretests to determine their achievement in mathematics, their mathematical reasoning power, their inductive reasoning power, and their critical thinking abilities.

After approximately fifteen weeks of instruction in its assigned method, each class was given posttests in each of the areas listed above. The final testing group consisted of 18, 22, and 23 students in the first, second, and third classes respectively. The discovery groups showed no significant gains over the lecture group in mathematical achievement as measured by a standardized test. However, differences in mathematical reasoning and inductive reasoning power, as well as positive attitude change toward mathematics, were in favor of the discovery groups. The general conclusions was that "the use of techniques to promote student discovery of concepts made a better teaching and learning situation (p. 5305)."

A current study on discovery and expository sequencing in elementary mathematics instruction was conducted by Worthen (1967). The problem was "to describe and compare two instructional methods in a naturalistic setting where the learning tasks and time sample approximated normal classroom conditions (p. 45)." The methods compared were classified as "a discovery method" and "an expository method." In the discovery method used in Worthen's study, verbalization of each concept or generalization was delayed until the end of the instructional sequence by which the concept or generalization was to be taught. In the expository method used in Worthen's study, verbalization of each concept or generalization was the initial step in the instructional sequence by which the concept or generalization was to be taught. It was hypothesized that the discovery method would produce superior results on retention and transfer tests in the selected mathematical concepts, on tests for transfer of heuristics, and on measures of attitude toward arithmetic.

The subjects in Worthen's study were fifth- and sixth-grade pupils in 16 classes from eight elementary schools. Eight teachers, one from each school, were selected on the basis of mathematical and general teaching competence, experience, and willingness to participate in the project. The selection of the teachers determined the selection of the sample since the subjects were pupils in the established classes of the teachers. Worthen's design called for the same content to be taught in each class for a two-month period prior to the study.

Although the procedure used to assign methods to classes was not reported in detail, Worthen did report that no significant differences were found between the treatment groups on pre-treatment measures including IQ, arithmetic computation skill, arithmetic problem solving ability, prior knowledge of the selected mathematical concepts, and pupil perception of teaching behavior. However, a significant difference was found between the discovery and expository groups on prior attitude toward arithmetic. Pupils in the expository group demonstrated significantly better attitudes toward arithmetic than pupils in the discovery group.

During the instructional period, four subsections of a concept knowledge test were administered at the completion of the corresponding subsection of the instructional materials. After the six-week instructional period, tests of concept transfer, transfer of heuristics, and pupil attitude toward arithmetic were administered. A concept retention test was administered twice to both treatment groups, once five weeks after instruction and once eleven weeks after instruction.

On the concept knowledge test, the expository group performed significantly better than the discovery group. However, the discovery group performed significantly better on the concept retention test and on the heuristics transfer tests. The discovery method also seemed to produce superior transfer of mathematical concepts, although

Worthen stated that this finding was somewhat tenuous. The results yielded by the attitude measures indicated no significant difference in the two treatments. Although one of the major criticisms of discovery approaches has been that they are inherently more time consuming than expository approaches, Worthen's study revealed no support for such claims.

As a result of his findings, Worthen listed the following implications for educational practice:

1. To the extent that pupil ability to retain mathematical concepts and pupil ability to transfer heuristics of problem solving are valued outcomes of education, discovery techniques of teaching should be an integral part of the methodology used in presenting mathematics in the elementary classroom.
2. To the extent that immediate recall is a valued outcome of education, expository instruction should be continued as the typical instructional practice used in the elementary classroom.
3. The present study also suggests that pupils' ability to transfer concepts will likely be increased in proportion to the degree to which discovery techniques are used in the classroom (Worthen, 1967, p. 30).

The studies cited so far in this review have, for the most part, either claimed the superiority of discovery approaches involving little or no guidance, or have claimed that no significant differences existed between such discovery approaches and other methods of teaching. There is evidence in the literature that methods providing more guidance to the learner might result in superior mathematical achievement.

Michael (1949) studied the relative effectiveness of two methods of teaching ninth grade algebra. Method A, which Michael called "inductive," stressed student discovery of fundamental principles and relationships. Method B, called "deductive," emphasized the use of authoritative statements of rules with extensive practice or drill.

The cooperation of fifteen ninth grade algebra teachers was enlisted, and one intact class per teacher was used in the study without any shifting or reassignment of students in an effort to match groups. The procedure used to assign methods to classes was not reported.

Three instruments constructed by Michael served as both pretests and immediate posttests to measure computation, generalization, and attitudes. Although the same basic textbook was used in both treatment groups, the method of presentation varied according to the specific instructions for each teaching method. The length of the experimental period was not reported, but it was indicated that it differed from class to class.

The data provided by the computation posttest revealed that the two methods produced approximately equivalent gains in the area of computation with the exception of the process of multiplication, in which Method A produced significantly greater gains. The results from the generalization test were significantly in favor of the deductive procedures of Method B. Neither method produced significant changes in the attitudes of the students toward mathematics in general. The over-all results of the study tended to favor the deductive procedures of Method B. It should be noted that no delayed posttest or retention test was given. The findings of Hendrix (1947), Sobel (1956), and Worthen (1967), suggest that students taught by Method A may have performed better on a delayed posttest than those taught by Method B.

Craig (1952) conducted an experiment to collect evidence on "the transferability of guided learning to tasks of different levels of difficulty and to study changes in dependence of transfer on initial ability that may be brought about by increasing the amount of guidance in discovery of the bases determining correct responses (p. 582)."

Four groups of recent college graduates, each group consisting of 30 men, were equated for initial performance on the learning material. The learning material consisted of verbal items which required the learner to select one word in a group of five which did not belong with the others. The four groups received different amounts of guidance in the form of cues, clues, and information about the relationships determining correct responses. A test based on the same principles of item organization employed in the learning materials was used as a pretest and as a posttest to measure transfer.

Among the conclusions reached by Craig were the following:

1. Irrespective of the difficulty level of the items to which transfer is measured, the amount of transfer or training increases as more and more clues are provided to aid discovery of the bases for correct responses.

2. In transfer as in learning, the average effectiveness of guidance by grouping situations according to common organizational principles, with or without information concerning the nature of the grouping, is increased several times by supplying learners with short statements of the common principles determining the grouping (Craig, 1952, p. 582).

The implications of Craig's research for mathematics teaching are that, in addition to organizing mathematical materials to be learned, the teacher should be liberal with suggestions designed to aid discovery of the principles of the organization.

In a later study, Craig (1956) attempted to "determine the effect of directing learner's discovery of established relations upon retention and the ability to discover new relations (p.233)." Craig hypothesized that increased direction of discovery activity brings about increases in learning without accompanying losses in retention or transfer.

The subjects for the experiment were two groups of 53 college students each, with the subjects being randomly selected from Craig's education courses during one semester. The groups were given a differential amount of direction to help them discover the bases for the solution of a series of learning situations. The learning situations involved selection of one word from a set of five which did not belong with the other four.

Three days following the 33-day learning sequence, a posttest for knowledge of relations and a posttest for measuring post-training ability to discover and use unlearned relations were administered. The group which received the greater direction, including short summary statements of the bases determining correct responses, learned more relations than the more independent group. The two groups improved about equally in their ability to solve problems organized upon unlearned bases. Craig advanced the following implications:

> This evidence indicates that teachers and experimenters should be liberal with information designed to assist learners in their discovery of principles. Large amounts of external direction now may help to insure that the learner will have an adequate background of knowledge to direct his future discovery (Craig, 1956, p. 234).

Kittell (1957) conducted an experiment whose purpose was "to determine the relative effects of three amounts of direction to learners in their discovery of established principles on transfer to differing situations and on retention of learned principles (p. 403)."

The sample consisted of 132 sixth-grade pupils who were randomly assigned to three treatment groups using a stratification based on high, medium, and low reading achievement classifications. The three treatments were differentiated by three different combinations of clues to the principles determining correct responses to five-word items having four words related by a principle and one word which did not belong. In the "Minimum direction" treatment, three items were grouped together and separated from other groups of items by spacing. Students in the Minimum group were told that each group of three items was based on an underlying principle, but a verbal statement of the principle was not given. In the "Intermediate direction" treatment, students were provided with all the information supplied the Minimum group plus a verbal statement of the principle involved. The "Maximum direction" treatment provided students with all the clues given the Intermediate group plus oral statements of the three correct responses for each group of items. Each treatment group was pretested prior to the training period with a test designed to measure knowledge of the principles.

After a five-week training period, posttests designed to measure knowledge, transfer, and retention of the principles were administered. The results indicated that the group receiving an Intermediate amount of direction learned and transferred as many or more principles than the groups receiving less or more direction. The group receiving an Intermediate amount of direction also retained a greater proportion of the learned principles than the other two groups.

Kittell stated that the following implications for learning-teaching situations were suggested by his experiment:

1. In addition to organizing the materials used in learning, teachers should aid pupil discovery by suggesting meaningful relationships on which learners may base discovery and by providing practice with those relationships.

2. Providing statements of underlying relationships without specifying answers fosters learning, retention, and transfer to different situations (Kittell, 1957, p. 403).

The final study to be reviewed was conducted by Kersh (1958). Kersh compared three methods of learning arithmetical rules to determine if "meaningful learning" was an adequate explanation for the superiority of learning by independent discovery.

The sample consisted of 48 college students from two sections of an Educational Psychology course who volunteered for the study. An equal number of students were randomly assigned to each of three treatment groups. One group of students, called the "no-help" group, was required to discover the arithmetical rules without help. A second group, called the "direct-reference" group , was given some direction in the form of perceptual aids, with accompanying verbal instructions which directed their attention to the perceptual aids. The third group,called the "rule-given" group, was told the rules directly and was given practice in applying them.

At the end of the learning period, which lasted a maximum of 90 minutes for each student, the "rule-given" group demonstrated superior ability to apply the rules to solve specific addition problems. However, after one month, the "no-help" group was superior to both of the other groups.

Kersh concluded that the motivation given to the "no-help" group to continue their efforts to learn and practice the rules was a more adequate explanation of their superior performance on the delayed posttest than any explanation in terms of "meaningful learning." Of particular interest is the fact that the more-directed method resulted in superior performance on the immediate posttest, whereas the less-directed method resulted in superior performance on the delayed posttest.

Summary of the Literature Review

The efficacy of various methods of teaching mathematics has been the subject of numerous theories and empirical studies. The theoretical proposals of committees, conference groups, and individual theorists have ranged from enthusiastic recommendations for independent student discovery to recommendations for extensive guidance and direction by the teacher. Less-directed approaches are purported to develop greater understanding, creative and independent thinking, increases in intellectual potency, intrinsic rewards, training in the heuristics of discovery, and aids to memory processing. More-directed methods are purported to reduce the needs for independent search, eliminate wrong hypotheses more effectively, and take less time than less-directed methods.

Empirical studies have not offered decisive evidence for the superiority of any one particular method of teaching mathematics. Henderson (1963) offers two reasons for the current state of indecisiveness concerning the relative effectiveness of teaching methods:

> One is that there have not been enough studies sampling the domains named (subject matter, teachers, students, schools) to generate much confidence in the findings. The second is that the findings are not always consistent even for a particular domain (Henderson, 1963, p. 1025).

Drawing upon the theoretical proposals and empirical research which have been reviewed, the present study was designed in an effort to resolve some of the contradictions found in the literature. The decision to include in the present study a method (Method D) having many of the characteristics of "unverbalized awareness" approaches was influenced by the theories of Beberman (1962) and Hendrix (1961), and by the research of Hendrix (1947), Sobel (1956), and Neuhouser (1965). The major contribution of Hendrix' (1947) study to the present study was the description and preliminary evaluation of an "unverbalized awareness" approach. The studies by Sobel (1956) and Neuhouser (1965) were more closely related to the present study in that they were conducted with eighth- and ninth- grade classes under classroom conditions. However, the present study used a larger sample than either of these studies and used a procedure for assessing the extent to which teachers did, in fact, teach by the prescribed methods.

The decision to include a method (Method E) with some of the characteristics of discovery approaches together with statements of the principles following a teacher-student development was influenced by the theories of the Cambridge Conference on School Mathematics (1963), Craig (1953), Gagne (1966), Kinsella (1965), and Rosskopf (1953), and by the research of Howitz (1966), Worthen (1967), Craig (1952, 1956) and Kittell (1957). The present study was most closely related to the studies by Howitz (1966) and Worthen (1967). In Howitz' study, content and method were confounded. Worthen controlled mathematical content for a two-month period prior to his experiment, but method of presentation was allowed to vary. In these respects, the present study differed from the studies of Howitz and Worthen. As pointed out in Chapter I, content was held constant in the present study while methods vary, and all participating classes were taught by the assigned experimental methods for an eight week period during the study.

The theories of Cronbach (1965) and Ausubel (1961) cast sufficient doubt on the notion that discovery appraoches are inherently superior to more-directed approaches to include a method (Method S) in the present study which emphasized teacher statement and application of mathematical principles as the initial steps in teaching new principles. The research findings of Michael (1949) and Nichols (1956) also influenced the decision to include Method S in the present study. The small treatment groups (21 students each) used by Nichols and the failure of Michael to administer a delayed posttest constituted the major differences in these studies and the present study.

CHAPTER III

PROCEDURES

Briefly, the procedures were as follows. Eighteen classes (approximately 400 students) were randomly assigned to three treatment groups--six classes per group. Each treatment group was taught three self-contained instructional units by its assigned method. Making statistical adjustments for initial differences in IQ, general mathematical achievement, and unit pretests among treatment groups, treatments were compared using unit immediate posttests and delayed posttests as criteria.

Subjects

Subjects were eighth and ninth grade general mathematics students. Through contacts with administrators and teachers in two Georgia public schools (designated "School A" and "School B"), the cooperation of seven general mathematics teachers was obtained. These teachers made available a total of 18 classes (approximately 400 students), 12 at the eighth grade level and 6 at the ninth grade level. On the basis of performance on the Arithmetic Concepts subtest of the Stanford Achievement Test and the Otis Quick-Scoring Mental Ability Tests, all participating classes were average or below average in general mathematical achievement and scholastic aptitude.

Treatments were randomly assigned to classes so that (1) each treatment was assigned to six classes--four eighth grade and two ninth grade, (2) each teacher taught by exactly two methods, (3) all three methods were used in both schools, and (4) each method was used in both morning and afternoon classes. Following the assignment of treatments to classes, each class was assigned a code number 1 through 18. Classes 1 through 6 were Method D classes, Classes 7 through 12 were Method E classes, and Classes 13 through 18 were Method S classes.

Subjects who failed to take the unit pretest or one of the posttests for any given unit were dropped from the data analysis for that unit. The number of students used in the analysis for Units 1, 2, and 3 were 392, 411, and 406,respectively.

Teachers

As stated above, seven general mathematics teachers participated in the study. These teachers were assigned code numbers 1 through 7. Table 1 shows for each class the code number of the teacher who taught the class, the school, the grade level, and the number of students in the class who completed each unit.

TABLE I

FOR EACH CLASS: THE CLASS CODE NUMBER, TEACHER, SCHOOL, GRADE LEVEL, AND NUMBER OF STUDENTS WHO COMPLETED EACH UNIT

Code Numbers of Classes in Treatments	Teacher	School	Grade Level	Number of Students Who Completed Each Unit		
				Unit 1	Unit 2	Unit 3
Method D (Classes 1-6)						
1	2	A	9	13	21	21
2	3	A	9	15	18	15
3	3	A	8	21	23	22
4	4	A	8	22	20	22
5	5	B	8	24	23	24
6	6	B	8	30	29	28
Method E (Classes 7-12)						
7	1	A	8	23	23	20
8	1	A	9	26	27	26
9	1	A	8	19	21	22
10	3	A	9	17	25	24
11	6	B	8	19	18	17
12	7	B	8	25	25	25
Method S (Classes 13-18)						
13	1	A	9	22	23	22
14	1	A	8	26	22	27
15	2	A	9	14	20	17
16	4	A	8	25	22	22
17	5	B	8	30	30	32
18	7	B	8	22	21	20

Table 2 shows, along with information on background and experience the pair of methods used by each teacher. Five different teachers used Method D, four used Method E, and five used Method S. The teachers within each treatment group held approximately equivalent college degrees, and all were certified in mathematics. In terms of the number of years of teaching experience in mathematics only, teachers in Methods D and S were more experienced than teachers in Method E. The teachers from School B were participating in an NSF in-service institute at the time of the study and were given an opportunity to become familiar with the Experiences in Mathematical Discovery (EMD) units as part of the institute course requirements.

TABLE 2

CODE NUMBER, TEACHING METHODS, SEX, COLLEGE DEGREE, AREAS OF CERTIFICATION AND YEARS OF TEACHING EXPERIENCE OF EACH TEACHER

Teacher Code Number	Teaching Methods Used	School	Sex	College Degree	Areas of Certification	Number of Years Teaching Experience	
						Total	Mathematics Only
1	E & S	A	Male	B.S.	Mathematics, Physical Education	4	2
2	D & S	A	Female	B.S.	Mathematics	5	5
3	D & E	A	Male	B.A.	Mathematics	1	1
4	D & S	A	Female	B.S.	Mathematics	3	3
5	D & S	B	Female	B.S.	Mathematics	8	8
6	D & E	B	Female	B.A.	Mathematics	2	2
7	E & S	B	Female	B.S.	Mathematics	2	1

In order to train the participating teachers in the experimental methods, two training sessions were held prior to the instructional phase of the study. The training sessions consisted of discussion of the major components of each experimental teaching method, demonstrations of each method, and special instructions pertaining to the administration of the testing instruments. A written description of the major components of each method was given to each teacher to serve as a reference during the study. Three additional training sessions were held, one during each unit.

Instructional Materials

In trying to select appropriate mathematics content for accomplishing the purpose of this study, several sets of published materials were reviewed. The series of units Experiences in Mathematical Discovery (EMD), published in 1966 by the National Council of Teachers of Mathematics, was considered particularly appropriate. The EMD units were designed for use by ninth grade general mathematics students, and a study by Howitz (reviewed in Chapter II) demonstrated their effectiveness. Four of the units--"Formulas, Graphs and Patterns," "Properties of Operations with Numbers," "Mathematical Sentences," and "Geometry" --provided the basis from which content was selected to write the instructional materials for the present study. Prior to the writing of the materials, permission was received from the National Council of Teachers of Mathematics to modify the EMD units. (See the letters of correspondence in Appendix A, p. 78 .)

Content selected from the EMD units was rewritten to conform to the definitions of the three teaching methods--Method D, Method E, and Method S. The resulting materials for Method D were presented as three separate units--Unit 1, Unit 2, and Unit 3. The same procedure was followed with the Method E and Method S materials. This procedure made possible a comparison of the three treatment groups at the completion of each unit.

Each student was furnished a set of instructional materials. The materials provided spaces for the student to record his name, and his responses to the questions in each lesson. Each unit contained nine lessons and most classes moved at the rate of one lesson per day.

The instructional materials were typed on off-set masters and reproduced in quantity using the off-set duplicating facilities at the College of Education, University of Georgia. Approximately 500 sets of materials were duplicated and delivered to the participating classes. Upon completion of instruction, the materials were returned to the experimenter.

Each teacher who participated in the present study was furnished a special set of instructional materials for each of his assigned teaching methods. The teacher's materials were identical to the student materials with the added feature of guiding statements which indicated how each section should be taught in order to stay within the definitions of the prescribed teaching methods.

Instruments

The following instruments were used in the study:

1. The Arithmetic Concepts subtest of the Stanford Achievement Test, Advanced Battery, Form X

2. The Beta Test of the Otis Quick-Scoring Mental Ability Tests, Form EM

3. Unit 1 Pretest, Immediate Posttest, Delayed Posttest, Student Rating Scale, and Observer Rating Scale.

4. Unit 2 Pretest, Immediate Posttest, Delayed Posttest, Student Rating Scale, and Observer Rating Scale.

5. Unit 3 Pretest, Immediate Posttest, Delayed Posttest, Student Rating Scale, and Observer Rating Scale.

6. Dutton's "Attitude Toward Mathematics" Scale

7. Fundamental Concepts in Arithmetic

The Arithmetic Concepts subtest of the Stanford Achievement Test was selected for the following reasons: (1) the test provides a measure of student achievement in arithmetic concepts which are commonly accepted as desirable outcomes of the elementary curriculum, (2) national norms are available for the test, and (3) the test has been used in other experimental studies and has been accepted by many researchers as a valid and reliable instrument.

The Beta Test of the Otis Quick-Scoring Mental Ability Tests was selected for the following reasons: (1) the test provides a measure of mental ability from which an IQ can be drived, (2) national norms are available for the test, and (3) test data for the students in one of the participating schools were available through the school's testing program.

Instruments listed in 3-5 above were constructed by the experimenters for use in the study.

Dutton's "Attitude Toward Mathematics" scale was selected for the following reasons: (1) the scale provides an objective measure of attitude toward mathematics, (2) the scale is appropriate for use by teachers and students, and (3) the scale has a high reliability coefficient (0.94) although it consists of only 15 items.

Fundamental Concepts in Arithmetic was selected to measure teacher understanding of arithmetic prior to the study.

Arithmetic Concepts Subtest of the Stanford Achievement Test

Administering the Arithmetic Concepts subtest of the Stanford Achievement Test made it possible to appraise the average achievement level of each participating class, as well as to determine the approximate percentile rank of each class in terms of national norms. Performance on this test was not treated as a covariate in the data analysis.

The Arithmetic Concepts subtest, one of three arithmetic tests in the Advanced Battery of the 1964 Stanford Achievement Test, is designed for use from the beginning of Grade 7 to the end of Grade 9. The test consists of 40 multiple-choice items, allows a working time of 25 minutes, and is scored objectively. The test yields a single raw score which can be translated into a grade score, percentile rank or stanine. In constructing the test, the authors sought to insure content validity by examining courses of study and textbooks to determine what concepts should be measured.

The test has a reported Kuder-Richardson Formula 20 reliability coefficient of 0.87 in Grade 8.6, and 0.88 in Grade 9.6. These values are based on a sample of 1000 cases for each grade drawn randomly from 76 school systems. The reported error or measurement at Grade 8.6 is 8.0, and at Grade 9.6 is 9.5 (Stanford Achievement Test, 1964, p. 11).

Bryan (1965), in a review of the arithmetic tests in the Advanced Battery of the 1964 Stanford Achievement Test, stated,

> ..."The Advanced Battery reflects more than the other batteries the influences of contemporary changes in the mathematics curriculum particularly in Test 5, Arithmetic Concepts, where the following topics are tested: divisibility, short cut computation by factoring, commutative and distributive properties,prime numbers, and numerations in other bases (p. 118)."

Bryan concluded by stating,

> ..."In providing a measure of that phase of the traditional mathematics curriculum known by the general term "arithmetic," the 1964 Stanford Achievement Test continues to be oustanding among tests of its kind (p. 118.)"

The Arithmetic Concepts subtest was administered to all participating classes by their respective teachers in November, 1967. Students marked their answers on IBM 1230 answer sheets, and the sheets were hand scored during December, 1967.

Beta Test of the Otis Quick-Scoring Mental Ability Tests

The Beta Test of the Otis Quick-Scoring Mental Ability Tests was used to provide a measure of mental ability from which an IQ could be derived. IQ was used as a covariate in the data analysis.

The Beta Test, published in 1954, is one of three Otis Quick-Scoring Mental Ability Tests, and is designed for use in Grades 4-9. The test contains 80 multiple-choice items, allows 30 minutes of working time, and yields a single score which can be converted to a "Mental Age" using a table provided by the publisher. A student's IQ can then be derived either by multiplying the quotient of his Mental Age and "chronological age" by 100 or by referring to the publisher's table. The latter method was used in this study.

The validity of the Beta Test was established by including only those items which definitely contributed to the capacity of the test to measure brightness as reflected in a student's rate of progress through school. The mean validity index of the items in the Beta Test is reported to be approximately 0.45. The odd-even reliability coefficient, corrected by the Spearman-Brown Formula, is reported to be 0.83 for Grade 6 and 0.85 for Grade 8. The standard error of measurement, based on a sample of 665 pupils is 4.8 (Otis Quick-Scoring Mental Ability Tests, 1954, p. 7).

Lefever (1959) indicated that the considerable emphasis given by the Beta Test to use of verbal symbols makes it particularly effective in predicting school success. He concluded:

> The Otis Quick-Scoring Mental Ability Tests, as the title implies, do furnish a short and easily scored indicator of scholastic aptitude. Such a measure, if interpreted with care, can be useful to both teacher and counselor by revealing within fairly broad limits of accuracy the probable level of academic achievement for a majority of pupils (Lefever, 1959, p. 488).

In one of the two schools which took part in the study, the Beta Test was administered by classroom teachers in January, 1968, to all participating classes. Students responded on IBM 503 answer sheets, which were later hand scored. In the other school which took part in the study, data on the Beta Test were available as a result of a Fall, 1967, administration of the test as part of the school's regular testing program. The data collected in the Fall administration were made available to the experimenters by school officials.

Instruments Constructed by the Experimenters

A total of 12 instruments were constructed by the experimenters for use in this study. Six of the instruments were used to assess student achievement or retention of the selected content. The remaining six were used to assess the extent to which teachers did, in fact, teach by the prescribed teaching methods.

Purposes and special characteristics of the instruments are discussed below.

1. The *Unit 1 Pretest and Delayed Posttest*, *Unit 2, Pretest and Delayed Posttest*, and *Unit 3 Pretest and Delayed Posttest* were used to measure pre-unit achievement and post-unit retention of the selected content in the respective units. The instruments were administered as pretests immediately preceding the respective units, and as delayed posttests approximately five weeks following the completion of the respective units.

2. The *Unit 1 Immediate Posttest*, *Unit 2 Immediate Posttest*, and *Unit 3 Immediate Posttest* were used to measure achievement of the selected content in the respective units. The instruments were administered immediately following the completion of the respective units.

3. The *Unit 1 Student Rating Scale*, *Unit 2 Student Rating Scale*, and *Unit 3 Student Rating Scale* were used to measure student conception of classroom procedure during the respective unit. These scales were completed every other day by two students in each class. The scales were analyzed both during and following each unit to give the experimenters a rough approximation of the degree of adherence to the prescribed teaching methods.

4. The *Unit 1 Observer Rating Scale*, *Unit 2 Observer Rating Scale*, and *Unit 3 Observer Rating Scale* were also used to gain a rough idea of the degree of adherence to the prescribed methods. These scales were used by two adult observers--one member of the mathematics department in each school. At least two observations were made of each teacher, one during each of his assigned methods. The analysis of the student and observer rating scales revealed that the teaching methods were not adhered to as closely as was desired.

Dutton's Attitude Toward Mathematics Scale

Dutton's "Attitude Toward Mathematics" Scale was used to measure both teacher and student pre-experimental and post-experimental attitudes toward mathematics. The scale was administered to teachers and students prior to Unit 1 and following Unit 3.

Fundamental Concepts in Arithmetic

Fundamental Concepts in Arithmetic, developed by the University of Georgia Mathematics Education Department, was administered to the seven participating teachers prior to the study in order to obtain a measure of teacher understanding of arithmetic. Achievement on this test indicated that each teacher was sufficiently knowledgeable in basic mathematics for participation in the study.

Methods of Data Analysis

Analysis of variance and analysis of covariance were the statistical techniques applied to the data.

Since each of the factors (1) prior general mathematical achievement, (2) mental ability, and (3) prior achievement in the selected content, was correlated with achievement in selected mathematics, the analysis of covariance was chosen as the primary statistical technique for Units 1 and 3. The analysis of variance was selected to test for first-order interactions between the main variables and as a supplementary analysis when the data were classified by each of the main variables.

The programs employed in the analysis of Units 1 and 3 were (a) a one-way analysis of covariance computer program developed by Cruz and Wilson (Wilson, 1967) and (b) a two-way analysis of variance computer program developed by Appelbaum and Bargmann (Bargmann, 1967) called Program MUDAID (Multivariate, Univariate, and Discriminant Analysis of Irregular Data). A test for homogeneity of regression equations was a feature of the Cruz and Wilson program that influenced its selection for the analysis of covariance. MUDAID was employed (a) as a test for first order interactions between the main variables and (b) as a supplementary analysis among group means of each of the main variables in analyses for which the analysis of covariance was inappropriate due to heterogeneity of regression.

The three variables that were the covariates for the analysis of covariance--(1) prior general mathematical achievement, (2) mental ability, and (3) prior achievement in the selected content--were each classified into three levels and treated as factors in the two-way analysis of variance. The 25th percentile and the 75th percentile were the values chosen a priori for determining the three classification groups of prior achievement. Consequently, the classifications described below come closest to satisfying these criteria. A subject was placed into the low, the average, or the high, prior general mathematical achievement group according to whether his raw score on the Arithmetic Concepts subtest of the Advanced Arithmetic Tests of the Stanford Achievement Test, Form X, was below 13, between 12 and 20, or above 19, respectively. A subject was placed into the below average, average, or above average group of mental ability according to whether his IQ score on the Otis Quick-Scoring Mental Ability Test, Form Beta, was below 90, between 89 and 111, or above 110, respectively. A subject was placed in the low, the average, or the high prior achievement group in the selected content of Unit 1, (Unit 3) according to whether his raw score on the Unit 1 pretest (Unit 3 pretest) was below 6 (below 10), between 5 and 9 (between 9 and 15), or above 8 (above 14).

The experimental designs used in Unit 2 were as follows: (1) a groups-within-treatments design, (2) a three-factor factorial design at the ninth grade level, and (3) a four-factor factorial design at the eighth grade level. The groups-within-treatments design, described by Lindquist (1956), regarded the class as the sampling unit, and treatments (methods) were randomly assigned to classes rather than individual students. The analysis of variance technique for the groups-within-treatments design, described by Lindquist (1956), was used to make treatment comparisons on achievement and retention in Unit 2.

The groups-within-treatments design in Unit 2 was supplemented by a three-factor factorial design at the ninth grade level and a four-factor factorial design at the eighth grade level. The three factors in the factorial design at the ninth grade level were "treatments," "classes," and "achievement level on the Unit 2 pretest." The four factors in the factorial design at the eighth grade level were "treatments," "schools," "classes," and "achievement level on the Unit 2 pretest." These supplementary designs provided an increase in the degrees of freedom and allowed an examination of main effects and interaction effects of "schools," "classes," and "Pretest-achievement-level" factors. The factorial designs were analyzed using an unweighted-means analysis of variance in which the student was treated as the unit of analysis. The cell frequencies in each factorial design were equalized using Winer's (1962) harmonic mean adjustment. The "achievement level" on the Unit 2 Pretest of a subject of a given group was high or low depending on whether he scored (a) on or above the group median or (b) below the group median of the Unit 2 Pretest.

The t-test was the statistical method employed to analyze the student and teacher attitude toward mathematics data.

The achievement tests constructed by the experimenters for each unit were analyzed by a computer program called TSSA2 (Wolf, 1962). The analysis of the rating scales is described in Chapter IV.

For each unit test, the pretest data, the Immediate Posttest data (achievement data), and the Delayed Posttest data (retention data) were analyzed by the computer program TSSA2 (Wolf, 1963). The reliability coefficients were as follows: (a) 0.32, 0.67, and 0.58 on the Pretest data of Unit 1, Unit 2, and Unit 3, respectively; (b) 0.74, 0.85, and 0.78 on the Immediate Posttest data of Unit 1, Unit 2, and Unit 3 respectively; and (c) 0.74,0.86, and 0.76 on the Delayed Posttest data of Unit 1, Unit 2, and Unit 3, respectively.

CHAPTER IV

RESULTS

The results of the present study are reported in five general sections. "Analysis of the Instruments" includes four subsections, presenting first the analyses of the Student Rating Scale (SRS) and the Observer Rating Scale (ORS) for Unit 1, presenting second the analyses of the SRS and the ORS for Unit 2, presenting third the analyses of the SRS and the ORS for Unit 3, and presenting fourth, the analyses of Test 1, Test 2, and Test 3. The data analysis are reported by unit in the general sections "Analysis of Unit 1," "Analysis of Unit 2," and "Analysis of Unit 3." The final general section is "Analysis of the Student and Teacher Attitude Data."

Analysis of the Instruments

Analysis of the Unit 1 Data Collected with the Student Rating Scale (SRS) and the Observer Rating Scale (ORS).

The procedure for the analysis of SRS for Unit 1 was to determine each student score on the SRS by summing algebraically the weights in Table 3 (Appendix C, p. 95) for the items marked by the student and determining the means of the student scores for each teacher-method group. The mean of each teacher-method group could then be compared with the highest possible score for each method--+3 for Method D, +4 for Method E, and + 3 for Method S -- and with the lowest possible score for each method -- (-6) for Method D, -4 for Method E, and -4 for Method S. The nearer the mean score of a teacher-method group approached the highest possible score for that method the stronger the confirmation of the teacher's fidelity to the method. Also, the nearer the mean score of a teacher-method group approached the lowest possible score for that method the weaker the confirmation of teacher fidelity to the method.

Table 4 (Appendix C, p. 96) presents the mean score and the number of observations for each teacher-method group during Unit 1. Teacher 1 received a mean rating of -0.2 on Method E and a mean rating of +1.1 on Method S. (Each teacher taught by two methods, thus one cell is empty in Table 4 for each teacher.) Data were unavailable on Teacher 2. Teacher 3 received a rating of -1.3 on Method D and + 1.3 on Method E. Teacher 4 received a rating of -0.1 on Method D and a rating of +0.6 on Method S. Teacher 5 received a rating of +1.3 on Method D and -0.2 on Method S. Teacher 6 received +0.6 on Method D and +0.2 on Method E. Teacher 7 received +2.2 on Method E and +2.8 on Method S. Notice that Teacher 7 showed stronger adherence to her assigned methods than any of the other teachers by student raters. Notice also that the means in Table 4 indicate a wide range of adherence from teacher to teacher within each method.

The teachers were also rated during Unit 1 by a mathematics teacher within his school. Each of these two observers agreed to observe each teacher-method group twice during Unit 1. However, the observer at School 1 found later that he had time for only one observation on each teacher-method group. The procedure of analysis of the data collected for Unit 1 on the Observer Rating Scale (ORS) was to add the weights of checked items as given in Table 3 (Appendix C, p. 95) and divide this sum by the number of items.

Table 5 (Appendix C, p. 97) presents the scores for Unit 1 on the ORS. Notice that Teachers 5, 6, and 7 have two scores per assigned method. The first of the two scores in each cell was for the observation during the first week of instruction and the second score was for the second observation at the end of the second week of instruction of Unit 1. Notice that Method S has the widest range of scores and Method D has the most consistent set of scores. Also, notice in Table 5 that the second rating of each of Teachers 5, 6, and 7, was at least as high as the score on the first observation and for Method S, there was a substantial increase from the first rating to the second rating for Teachers 5 and 7. The data indicated that strongest adherence was to Method D and the weakest adherence was to Method S during Unit 1.

Analysis of Unit 2 Data - Collected with SRS and ORS

The ORS for Unit 2 was analyzed by computing proportions of responses that indicated adherence to the prescribed teaching methods. Part I of the ORS for Unit 2 Scale contains six items, and Part II contains one item. Thus, each time a rater observed a class he responded to seven items on the basis of his observation. The rater's response to each item was recorded by the experimenter as indicating either "adherence" or "non-adherence" to the prescribed teaching method. The proportion of confirming responses (C_p) for each rating was computed by summing the number of responses which indicated adherence to the prescribed method and then dividing this sum by the total number of responses, namely 7.

The design of the study called for a total of two observer ratings on each teacher, one rating in each of the teacher's assigned teaching methods. The design was adhered to except in one instance; the observer in School A was unable to observe Teacher 4. Table 6 (Appendix C, p. 98) shows the proportion of confirming responses (C_p) on the ORS for each of the two teaching methods of each teacher who was observed. Since the results in Table 6 are based on only one observation each, they should not be considered as definitive of the degree of adherence to the prescribed methods during the entire study. They are intended. rather, as a rough approximation of the degree of adherence during the periods of observation. Table 6 indicates weaker adherence in Classes 9, 10 and 17 and stronger adherence in Classes 5, 6, 11, 15, and 18.

Table 7 (Appendix C, p. 99) shows the proportion of confirming responses (C_p) on the ORS for each treatment group. The C_p for each treatment group was computed by summing the number of responses within each treatment group which indicated adherence and then dividing this sum by the total number of responses within a treatment group, namely 28. Table 7 indicates weakest adherence to Method E and strongest adherence to Method D. Adherence to Method S fell between the other two methods. These results should be interpreted with the same caution that was pointed out above.

Proportions of confirming responses were computed separately for the two parts of the Student Rating Scale for Unit 2. Part I of this instrument contains nine items which deal with specific components of the teaching methods, and Part II contains one item which deals with a global, or general, description of each teaching method. The design of the study called for two student raters in each class to respond to the instrument on four separate occasions. Thus, for those classes in which the design was adhered to, student raters responded to a combined total of 72 items on Part I and a combined total of 8 items on Part II. The experimenter recorded each response as indicating either "adherence" or "non-adherence" to the prescribed teaching method. The proportion of confirming responses (C_p) on Part I for the two raters in a given class was computed by summing the number of responses which indicated adherence to the prescribed method and then dividing this sum by the total number of responses. The proportion of confirming responses (C_p) on Part II was computed in a similar way.

To obtain a measure of rater agreement on the global, or general, item which made up Part II of the SRS, an "agreement proportion" (A_p) was computed for each pair of student raters. For a given pair of raters, A_p was computed by counting the number of occasions on which both raters indicated adherence or both indicated non-adherence and then dividing this total by the number of rating occasions.

Table 8 (Appendix C, p. 100) shows the proportion of confirming responses (C_p) on the SRS for the pair of student raters in each class. The table also shows the agreement proportion (A_p) for the pair of student raters in each class. It should be kept in mind that the results shown in Table 8 are not to be considered as definitive of the degree of adherence to the prescribed methods during the entire study. The results are intended as a rough approximation of the classroom procedure followed in a given class as seen by two particular students (the two student raters picked by the teacher). Table 8 indicated weaker adherence on Part I for Classes 1, 2, and 11 and stronger adherence for Classes 5, 6, 12, 13, 14, 15, and 18. On Part II, weaker adherence was indicated for Classes 7, 8, 9, and 14 and stronger adherence was indicated for Classes 2, 5, 6, 13, and 18. Table 13 also shows that there was less agreement on Part II between the pairs of raters in Classes 4, 10, 12, 16, and 17 and more agreement between the pairs of raters in Classes 2, 5, 6, 9, 13, and 18.

Table 9 shows that the proportion of confirming responses (C_p) on the two parts of the SRS for each treatment group. The C_p for each treatment group on each part of the SRS was computed by summing the number of responses within each treatment group which indicated adherence and then dividing this sum by the total number of responses within a treatment group varied from group to group since some of the pairs of raters observed on less than four occasions. Table 9 (Appendix C, p. 102) indicated weakest adherence to Method E and strongest adjerence to Method S on Part I. On Part II, Table 9 indicates weakest adherence to Method E and strongest adherence to Method D. As pointed out above, these results are only rough approximations and are not to be considered as definitive of the classroom procedures which were actually followed during the entire study.

Analysis of Unit 3 Data Collected with SRS and ORS

Both the Student Rating Scale for Unit 1 and the Observer Rating Scale for Unit 1 were revised for use during Unit 3. The frequence of qualifying remarks to checked items on these scales during Unit 1 was the prime reason for the revisions. The comments and remarks suggested the need for options allowing different degrees of agreement. The revised Student Rating Scale consists of fifteen items each of which corresponds to the same item on the revised Observer Rating Scale except for item number 13. Also, the ORS contains one more item than the SRS.

The procedures for the analyses of the data collected with the SRS and the ORS for Unit 3 was analgous to the procedures for Unit I. Tables 10 and 11 (Appendix C, pp.103,104) give the tables of weights from which scores were computed on the SRS and the ORS respectively. Briefly, each student score on the SRS was the quotient of the algebraic sum of weights from Table 10 of the options marked by the student and the number of items (15). The arithmetic mean of the student scores was then the score indicating the degree of conformity of the teacher to his assigned method. The nearer the mean score to the maximum score possible in each method (+2 for each method), the stronger the confirmation of adherence by the teacher and the closer the mean score to the minimum score possible for each method (-2 for Method D, -1.93 for Method E, and -1.87 for Method S) the stronger the confirmation of non-adherence to the teaching method.

Table 12 (Appendix C, P. 105), presents the number of observations and the mean on the SRS for each teacher on his assigned methods. Notice that all scores in Table 12 are positive, with some scores indicating strongest adherence to Method D.

Table 13 (Appendix C, p. 106) presents the ORS scores obtained from Table 11 in the same manner as SRS from Table 10, with almost identical limits) for each teacher on each of his assigned methods. The data shows that all scores were positive, thus tending toward confirmation of adherence to the methods. Notice that all scores for Method D, regardless of the teacher, received the strongest confirmation of adherence. Thus, these results supported the results of the analysis of the SRA data in Unit 3.

Unit 1 Results

The results of Unit 1 are reported in the following manner: (1) those analyses for which the F-values were not significant at the .05 level in group mean comparisons and (2) those analyses for which the F-values were significant at the .05 level in group mean comparisons and in selected first-order interactions.

Analyses for Which F-values Were Non-Significant in Group Comparisons

When the criterion was achievement in Unit 1, there were no significant differences among the classification group means when the data were stratified by (1) student sex, (2) grade level, (3) period of day, (4) treatment (method) within the subset of male students, (5) treatment within School 2, (6) treatment within the subset of eighth grade boys, (7) treatment within the subset of ninth grade boys, (8) treatment within the subset of male students of female teachers, and (9) treatment within the subset of female students of male teachers. Results of the analyses under stratifications (7) and)9) above should be regarded tennous due to significant heterogeneity of regression at the .05 level. The results of these analyses are summarized in Table 14 (Appendix C, p.   ) giving the adjusted means for each group. When the criterion was retention in Unit 1, there were no significant differences among classification group means when the data were stratified by (1) student sex, (2) grade level, (3) treatment within the subset of male students, (4) treatment within the subset of eighth grade male students, (5) treatment within the subset of ninth grade male students, (6) treatment within the subset of male students of female teachers, and (7) treatment within the subset of male students of male teachers. A summary of the results of the preceeding analyses of the retention data of Unit 1 is given in Table 15 (Appendix C, p. 108) including the adjusted mean for each comparison group.

Analyses for Which F-Values Were Significant

Each analysis for which the F-value was significant in group comparisons and first-order interactions is reported in this section. For each stratification of the data the results are reported first when the criterion was achievement and second when the criterion was retention. Significant first-order interactions, at the 105 level, are reported when either of the main variables involved is first used as a classification variable in group comparisons.

Stratification by Treatment Method.

Table 16 (Appendix C, p. 109) presents the analysis of covariance of achievement in Unit 1 when the data were classified by method. The analysis showed significant differences at the .01 level among the treatment means. However, heterogeneity of regression was also significant at the .01 level. The latter results indicated that the important basic assumption of homogeneity of regression in the analysis of covariance technique was highly doubtful when the data were stratified by method and the criterion was achievement in Unit 1. Thus, analysis of covariance was probably inappropriate for this analysis. So, the analysis of variance was applied to the data of Unit 1 with achievement as criterion when the data were grouped by the main variable method (Table 17, Appendix C. p 110). Table 17 reveals highly significant differences ($p < .01$) among the teaching methods and Table 18 (Appendix C, p. 111) shows the results of the range tests where M_D, M_E, and M_S represent the means of Method D, Method E, and Method S. Notice that while Methods E and S did not significantly differ, each was superior to Method D.

When the criterion was achievement in Unit 1, a significant interaction at the .01 level was found between method and teacher (Table 19, Appendix C, p. 112). A significant interaction at the .05 level was found between method and each of mental ability level and the Unit 1 pretest (Tables 20 and 21, Appendix C, pp.113-4). Figure 1(Appendix D, p. 194) illustrates graphically the interaction found between teacher and method. Notice in particular that the Method E and the Method S group means interchanged relative positions from teacher to teacher. However, before any hasty conclusions are drawn concerning the effectiveness of certain teachers with certain methods, it should be pointed out that classes were grouped according to ability before the experimental study was begun.

Figure 2 (Appendix D, p. 199 illustrates the interaction between method and mental ability. For the below average ability group the Method D subgroup mean was lowest, the Method subgroup mean highest, and the Method E subgroup mean in between the other two. For the average ability group, the Method D subgroup mean was still lowest, but the other two subgroup means were about equal. For the above average ability group, the Method E and Method S group means exchanged relative positions, and the Method D group mean remained in the lowest relative position. These results indicated that brighter students required less guidance than the average and below average ability students, but they performed best with some degree of guidance in learning.

Figure 3 (Appendix, D, p. 196) presents the interaction between method and prior achievement in Unit 1. These results indicated that when the students knew very little about the content prior to the instructional period, a moderate degree of guidance was more effective than an extreme degree of guidance. If the students were a little more knowledgeable of the content before the instructional phase of the study, any greater degree of guidance was slightly better than the minimum amount received in Method D. For the most knowledgeable group of students in the content of Unit 1 prior to the instructional phase of the study, the greater the degree of guidance the greater was the achievement.

Table 22 (Appendix C, p. 115) is a summary of the analysis of covariance of Unit 1 retention data when the total population was stratified by treatment. The results showed that there were significant differences at the .01 level among treatment group means on retention of Unit 1. The F-value for heterogeneity of regression was non-significant, thus supporting the basic assumption of analysis of covariance.

A summary of the range test is reported in Table 23 (Appendix C, p. 116) giving the adjusted means of each treatment group. As on achievement, the results showed that while the means for Methods E and S did not significantly differ, each was significantly greater at the .01 level than the Method D mean.

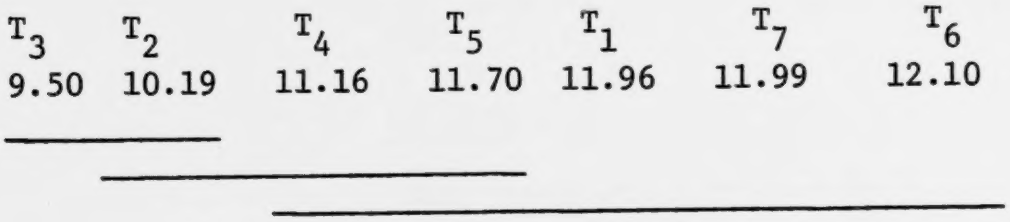
On retention of Unit 1 with the total population in the comparison there were significant interactions at the .01 level between method and teacher and between method and mental ability. Figure 4 (Appendix D, p. 197) illustrates graphically the interaction effect observed between method and teacher on retention of Unit 1. Notice that the relative positions of Method D and Method E group means interchanged from Teacher 3 to Teacher 6. This indicated that method effects were not independent of teacher effects. However, no conclusions should be drawn from these results about the effectiveness of each of these two teachers with a particular teaching method since the various teacher-method groups were originally grouped according to mathematical ability by the participating schools.

Figure 5 (Appendix D, p. 198) presents graphically the interaction between method and mental ability level. The results indicated that the effect of mental ability level on retention of Unit 1 was not independent of the method by which the unit was taught. The results suggested that for the low IQ group, the Method D subgroup retained the least and the Method S group retained the best. Retention by the Method D and Method E average mental ability groups did not significantly differ but each was lower than the retention for the Method S average mental ability group. For the high mental ability subjects the Method E subgroup had highest retention with the Method D subgroup retaining the least. Thus, the Method E and Method S subgroups had parallel positions for the average and below average subjects with retention by the Method S subgroups superior. However, the relative positions of the Method E and Method S subgroups reversed for the above average subjects.

This suggested that the bright students retained better when given a moderate degree of guidance whereas the less gifted students retained better with a maximum degree of guidance.

Stratification by Teacher

Table 24 (Appendix C, p.117) presents the analysis of covariance of the Unit 1 achievement data when the subjects were grouped by teacher. The analysis showed significant differences, at the .01 level, among adjusted teacher group means. Heterogeneity of regression was marginal at the .05 level, but not significant. A posteriori tests were performed on the adjusted means resulting from the analysis of covariance. These results, summarized in Table 25 (Appendix C, p.118), showed a significant difference (a) at the .01 level between the adjusted mean of the Teacher 3 group and each of the adjusted means of the Teacher 1 group, the Teacher 5 group, the Teacher 6 group, and the Teacher 7 group, and (b) at the .05 level between the adjusted group mean of Teachers 1, 6, and 7. If the means that did not significantly differ were considered, the following diagram summarizes the results:



In the diagram, means with a common underline did not significantly differ.

There was a significant interaction at the .01 level between teacher and mental ability level for achievement in Unit 1. Figure 6 (Appendix D, p. 199) gives a graphic illustration of the interaction effects between teacher and mental ability level. Notice that the mean scores of the three ability level groups were in the expected relative order (i.e., the high IQ group scored higher than the average IQ group which, in turn, scored higher than the low IQ group) for Teachers 1, 3, 4, 6, and 7 but not for Teachers 2 and 4. Under Teacher 2, the low mental ability group achieved as well as the average mental ability group and the high mental ability group achieved no better than the average mental ability group in Unit 1. Under Teacher 5, the mean scores of the average IQ group and the above average IQ group were in the expected relative order to each other. However, the mean score of the low IQ group was greater than the mean of either of the other two IQ groups. Thus, it appeared that achievement in Unit 1 was not due to independent effects of mental ability level and teacher. Due to the small number of cases in some cells, caution should be exercised in drawing conclusions at this point. There were 8, 15, and 4 subjects in the low IQ group, average IQ group, and above average IQ group, respectively, of Teacher 2. The analysis reveals a mean score of 15 and SD of 0 for the low IQ group with 2 subjects for Teacher 5, thus indicating a score of 15 per subject. It is conceivable that this mean score was due largely to chance.

Table 26 (Appendix C, p. 119) summarizes the analysis of covariance of Unit 1 retention data when the subjects were stratified by teacher. However, since heterogeneity of regression was significant at the .01 level, the analysis of variance in Table 27 (Appendix C, p. 120) will not be reported. This table summarizes the two-way analysis of variance of teacher by achievement level group means. These results showed significant differences at the .01 level among teacher group means and no significant interaction effects between achievement level and teacher. Table 28 (Appendix C, p 121)gives the paired comparisons between the teacher groups. In Table 28, the symbol T_i represents the adjusted mean retention score on Unit 1 by the subgroup of subjects taught by Teacher I, where i - 1, 2, ..., 7. The adjusted mean of the Teacher 7 group was superior at the .01 level to all other group means except that of the Teacher 5 group. The teacher group means partition into two sets as shown schematically in the following diagram which lists teacher groups from high to low:

$$\begin{array}{ccccccc} T_7 & T_5 & T_6 & T_4 & T_1 & T_2 & T_3 \\ 14.07 & 12.95 & 11.73 & 11.55 & 11.34 & 9.65 & 9.52 \\ \cline{1-2}\cline{3-5}\cline{6-7} \\ \cline{2-4} \end{array}$$

Means of those teacher groups with a common underline did not significantly differ at the .05 level. Means differed significantly for those groups without a common underline. Thus Teacher 5 and Teacher 7 group means did not significantly differ; Teacher 1, Teacher 4, and Teacher 6 groups did not significantly differ; and Teacher 2 and Teacher 3 group means did not significantly differ.

As shown in Table 29 (Appendix C, p 122), retention of Unit 1, there was a significant interaction at the .05 level between teacher and mental ability. This interaction is shown graphically in Figure 7. (Appendix C, p.). Notice that the effects upon retention of Unit 1 by the mental ability level of the group was not independent of the effects of the teacher. In particular, the effects of Teachers 2 and 5 were unlike those of the other teachers since their ability groups retained equally well. However, the reader should be reminded again of the small number of cases in the various ability groups of these two teachers.

Stratification by Period of Day.

Although no significant differences at the .05 level existed among group means for achievement in Unit 1 as shown in Table 30 (Appendix C, p. 123), there were significant differences at the .01 level among period group means when the cirterion was retention of Unit 1. Table 31

(Appendix C, p. 124) is a two-way table that summarizes the paired comparisons of group means. The group mean of period 6 was significantly different at the .01 level from each of the group means of periods 1 and 4. At the .05 confidence level, the period 1 group mean differed from each of the means of periods 2, 3, and 5, and the period 4 group mean differed from each of the group means of periods 2 and 5.

Stratification by School.

Table 32 (Appendix C,p.125) presents the analysis of covariance of the Unit 1 achievement data under the classification of school. At the .05 level there was a significant difference between school means and significant heterogeneity of regression. However, analysis of variance revealed significant differences at the .01 level. Thus, the school 1 group mean of 11.02 was inferior to the School 2 group mean of 12.01.

Table 33 (Appendix C, p. 126) presents the analysis of covariance of the Unit 1 data when the subjects were classified by school and the criterion was retention. The F-value for school mean differences was very large (F=25.797). The adjusted means were 10.85 and 12.82 for School 1 and School 2, respectively.

Stratification by Method Within the Eighth Grade

Table 34 (Appendix C, p. 127) presents the analysis of covariance of the Unit 1 achievement data when the eighth grade students were classified by method. The results of the analysis indicated highly significant differences among the treatment groups (at the .01 level). Heterogeneity of regression was marginal at the .05 level, casting some doubt on the validity of the analysis. With this restriction the results of the a posteriori tests are summarized in Table 35 (Appendix C, p. 128). The results show that there was no significant differences among the treatment groups (at the .01 level). Heterogeneity of regression was marginal at the .05 level, casting some doubt on the validity of the analysis. With this restriction the results of the a posteriori tests are summarized in Table 35 (Appendix C, p.   ). The results show that there was no significant difference between the means of the Method E group (12.21) and the Method S group mean (12.30) but that each of these was significantly greater at the .01 level than the Method D group mean (10.59).

Table 36 (Appendix C, p. 129) presents the analysis of covariance of the Unit 1 retention data of the eighth grade subjects when they were classified by method. The analysis showed significant differences at the .01 level among the method group means. Again, while the Method E and the Method S groups did not significantly differ, each was superior to the Method D group on retention at the .01 level.

Stratification by Method Within the Ninth Grade

Table 37 (Appendix C, p. 130) presents the analysis of covariance of the Unit 1 achievement data of the ninth grade subjects when grouped by method. Although the F-value for treatment means indicated significant differences among the method group means at the .05 level, heterogeneity of regression was significant at the .01 level. The latter fact indicated that the analysis of covariance was inappropriate for this stratification of the data.

Table 38 (Appendix C, p.131) presents the analysis of covariance of the Unit 1 retention data of the ninth grade subjects when they were grouped by method. The results showed that there were significant differences among the treatment means at the .05 confidence level while heterogeneity of regression was non-significant at the same level. Further analysis showed that there was a significant difference between Method D compared to Method E and Method S.

Stratification by Method Within the Female Teacher Subgroup.

Table 39 (Appendix C, p. 132) presents the analysis of covariance of the Unit 1 achievement data when the subjects were grouped by method within the female teacher subgroup. As seen from the table, there were significant differences at the .01 level among the treatment groups. Also, note that heterogeneity of regression was non-significant at the .05 level indicating that the basic assumption of homogeneity of regression was satisfied. The results of further analysis in binary comparisons showed that Method E differed from Method D at the .01 level while Method S differed from Method D at the .05 level.

As shown in Table 40 (Appendix C, p. 133), the F-value was even larger for treatment means when the criterion was retention of Unit 1 by the subjects of female teachers. Further analysis showed that while Method E and S did not significantly differ, each was superior to Method D at the .01 level.

Stratification by Method Within the Male Teacher Subgroup

Table 41 (Appendix C, p. 134) presents the analysis of covariance of the Unit 1 achievement data when the subjects were grouped by method within the male teacher subgroup. The table shows significant differences at the .01 level among the treatment means. However, the results of this analysis may invalid since heterogeneity of regression was significant at the .05 level. Table 42 (Appendix C, p. 135) presents the analysis of covariance of the Unit 1 retention data when the subjects were grouped by method within the male teacher subgroup. The results reveal significant differences among the treatment means at the .05 level and homogeneity of regression. The a posteriori test revealed a significant difference between Method D and Method E and between Method D and Method S at the .05 level but no significant difference between Method E and Method S.

Stratification by Method Within the Subset of Female Students.

Tables 43 (Appendix C, p.136) and 44 (Appendix C, p. 137) present the analyses of covariance of the Unit 1 achievement and retention, respectively, when the subjects were grouped by method within the female student subgroup. In each analysis there were significant differences at the .01 level among the treatment means and no significant heterogeneity of regression at the .05 level. The range tests also revealed identical results--there was a significant difference at the .01 level between Method D and Method E and between Method D and Method S but there was no significant difference between Method E and Method S in either case.

Stratification by Method Within School 1. Table 45 (Appendix C, p. 138) presents the analysis of covariance of Unit 1 achievement data when the subjects were grouped by method within School 1. The results showed significant differences at the .01 level among treatment means. However, there were also significant differences at the .01 level among regression equations.

Table 46 (Appendix C, p. 139) presents the analysis of covariance of Unit 1 retention data when the subjects were grouped by method within School 1. The results revealed significant differences among the treatment means at the .05 level and no significant heterogeneity of regression. Binary comparisons revealed a significant difference between Method D and Method E and between Method D and Method S at the .05 level but no significant difference between Method E and Method S.

Stratification by Method Within School 2.

Although there were significant differences at the .05 level among neither treatment means nor regression equations on achievement in Unit 1 within School 2, there were significant differences among treatment means at the .01 level and among regression equations at the .05 level when the criterion was retention. Table 47 (Appendix C, p.140) summarizes these results.

Stratification by Method Within the Eighth Grade Female Student Subgroup.

Table 48 (Appendix C, p. 141) presents the analysis of covariance of the Unit 1 achievement data when the eighth grade female subjects were grouped by method. There were significant differences at the .01 level among treatment means and no significant differences among regression equations. A posteriori tests revealed a significant difference at the .05 level between Method D and Method E, a significant difference at the .01 level between Method D and Method S, and no significant difference between Method E and Method S.

Table 49 (Appendix C, p. 142) presents the analysis of covariance of the Unit 1 retention data when the eighth grade female subjects were grouped by method. The findings were identical to those for achievement above.

Stratification by Method Within the Ninth Grade Female Student Subgroup

Table 50 (Appendix C,p.143) presents the analysis of covariance of the Unit 1 achievement data when the ninth grade girls were grouped by method. The results showed significant differences at the .01 level among both treatment groups and regression equations.

Table 51 (Appendix C, p.144) presents the analysis of covariance of the Unit 1 retention data when the ninth grade girls were grouped by method. The results revealed significant differences at the .05 level among treatment means and no significant differences among regression equations. Further analysis showed a difference at the .05 level between Method D and Method E only.

Stratification by Method Within the Subset of Female Students of Female Teachers

Table 52 (Appendix C, p. 145) presents the analysis of covariance of the Unit 1 achievement data when the girls taught by female instructors were grouped by method. The results showed significant differences at the .01 level among treatment means and no significant differences among regression equations. The range tests yielded a significant difference at the .05 level between Method D and Method E, a significant difference at the .01 level between Method D and Method S, and no significant difference between Method E and Method S.

Table 53(Appendix C, p 146) presents the analysis of covariance of the Unit 1 retention data when the girls taught by female instructors were grouped by method. The results showed significant differences at the .01 level among treatment means and no significant differences among regression equations. The range tests yielded a significant difference at the .01 level between Method D and Method E and between Method D and Method S, and no significant difference between Method E and Method S.

Stratification by Method Within the Subset of Female Students of Male Instructors

The analysis of covariance of the Unit 1 achievement data when the girls taught by male instructors were grouped by method indicated no significant differences among treatment means, as reported above. Also, heterogeneity of regression was marginal at the .05 level. However, there were significant differences among treatment means, with no significant heterogeneity of regression, when the criterion was retention. Further analysis revealed that the only significant difference was between Method D and Method E.

Stratification by Method Within the Subset of Male Students of Male Instructors.

Table 55 (Appendix C, p. 148) presents the analysis of covariance of the Unit 1 achievement data when the boys taught by male instructors were grouped by method. The results indicated that there were significant differences at the .05 level among treatment means and among regression equations. As seen previously, there were significant differences among neither the treatment means nor regression equations at the .05 level when the criterion was retention.

Unit 2 Results

The results of the analysis of Unit 2 are reported in the sections "Analysis of the Groups-Within-Treatments Design" and "Analysis of the Factorial Designs".

The analysis of variance technique for the groups-within-treatments design described by Lindquist (1956) was used to make treatment comparisons on achievement and retention in Unit 2. Before using the analysis of variance technique, a test for homogeneity of variance was made. The test for homogeneity of variance is reported first in this section, followed by the results of the analysis of variance.

Test for Homogeneity of Variance

A basic assumption in the analysis of variance for the groups within-treatments design is that the variance of the distribution of group means is the same for each treatment population. Before making the analyses on the achievement data and the retention data, Leven's Test as described by Glass (1966) was used to test for homogeneity of variance.

Levene's test was applied in this study by making a one-way analysis of variance on the absolute values of the differences between each class mean and the unweighted mean of its treatment group. Table 56 (Appendix C, P. 149) shows the results of the homogeneity of variance test on the achievement data. The obtained F-ratio was not significant at the .05 level, which indicated that the data supported the hypothesis of equal treatment population variances.

Table 57 (Appendix C, p 150) shows the results of the homogeneity of variance test on the retention data. Again, the F-ratio was not significant at the .05 level, which indicated that the hypothesis of equal treatment population variances was supported by the data.

Analysis of Variance for the Groups-Within-Treatments Design on the Achievement Data

Table 58 (Appendix C, p. 151) summarizes the results of the analysis of variance for the groups-within-treatments design on the achievement test. The obtained F-ratio (0.07) did not exceed the critical value of $F_{.95}$ (3.68).

Analysis of Variance for the Groups-Within-Treatments Design on the Retention Data

Table 59 (Appendix C, p. 152) summarizes the results of the analysis of variance for the groups-within-treatments design on the retention test. The obtained F-ratio (0.14)did not exceed the critical value of $F_{.95}$ (3.68)

Analysis of the Factorial Designs

It was pointed out in Chapter III that the groups-within-treatments design was supplemented by a factorial design at each grade level in unit 2. The factorial designs treated the student rather than the class, as the unit of analysis. As explained in Chapter III, "achievement level on the Unit 2 pretest" was treated as a factor in the factorial designs employed in this study. The use of this factor necessitated the division of each class into to sub-classes-- the subclass of students who scored at or above the overall median on the Unit 2 pretest (the overall median on the Unit 2 Pretest was 18). The letter "U" denotes the set of all students who scored at or above the overall median, and the letter "l" denotes the set of all students who scored below the overall median. Results of the analysis of variance for the factorial designs are reported in this section.

Analysis of Variance for the Three-Factor Factorial Design at the Ninth Grade Level on the Achievement Test

In the analysis of the three-factor factorial design at the ninth grade level, "treatments" (T), "classes" (C), and "achievement level on the Unit 2 pretest" (A) were treated as fixed, random, and fixed factors respectively. The 12 subclasses which resulted from blocking on these factors were considered to have $\bar{n}_h$ observations each, where $\bar{n}_h$ was the harmonic mean of the actual number of students in each subclass. The value of $\bar{n}_h$ used in the ninth grade analysis was 10.

Table 60 (Appendix C, p 151) summarizes the analysis of variance for the three-factor factorial design at the ninth grade level on the Unit 2 achievement test. The obtained F-ratio for the treatments factor did not exceed the critical value of $F_{.95}$. This indicated that at the ninth grade level the differences among treatment means on the Unit 2 achievement test were not significant at the .05 level.

The F-ratios for TA (interaction of treatments and achievement level on the pretest) and AC:T (interaction of achievement level on the pretest and classes) did not exceed the critical value of $F_{.95}$. The nonsignificant interaction indicated that the effect of a treatment did not differ for the two achievement levels, nor did the effect of a class differ for the two achievement levels.

The F-ratio for the classes factor exceeded the critical value of $F_{.95}$. This indicated that some of the differences among class means (or totals) were significant at the .05 level of significance. To determine which particular class totals differed significantly, the Newman-Keuls method was used to make tests on all possible pairs of ordered totals.

The Newman-Keuls method, described by Winer (1962), involves the calculation of a q_r statistic, where r is the number of steps two means (or totals) are apart on an ordered scale. When making a large number of tests, Winer recommends that a critical value for the difference between two totals which are r steps apart on an ordered scale is $q_{1-\alpha}(r,f)\sqrt{nMS_{error}}$ (α refers to the desired level of significance, f refers to the degrees of freedom for MS_{error}, and n refers to the number of subjects in each group whose means are being compared). The Newman-Keuls procedure keeps the level of significance equal to α for all ordered pairs, no matter how many steps apart the totals may be. However, the level of significance with respect to the collection of all tests made, considered as a single test, is lower than α. This is in contrast to Duncan's (1955) procedure which uses a protection level of α for the collection of tests, rather than a α level for the individual tests. The Newman-Keuls method is less powerful, but more conservative, than Duncan's procedure.

In the present analysis, the Newman-Keuls test was applied to class totals with the number of students in each class considered to be $2\bar{n}_h$, where $\bar{n}_h$ was the harmonic mean of the actual number of students in each pretest-achievement-level subclass.

Table 61 (Appendix C, p 152) shows the ninth grade class totals on the Unit 2 achievement test arranged in increasing order of magnitude. Each class total was computed by finding the product of $\bar{n}_h$ and each subclass mean, and then adding these two products.

The results of the Newman-Keuls test at the .05 level of significance may be summarized schematically as follows:

Class 2   Class 13   Class 1   Class 10   Class 15   Class 8

Classes underlined by a common line did not differ from each other; classes not underlined by a common line did differ. Thus, Class 8 differed from Class 2 and Class 13, but did not differ from Classes 1, 10 and 15. Similarly, Class 15 differed from Class 2, but Class 15 did not differ from Classes 13, 1, 10 and 8. There were no two classes in the same treatment that differed from each other.

Another result of the analysis of variance for the three-factor factorial design at the ninth grade level on the Unit 2 achievement test was that the F-ratio for the factor "achievement level on the Unit 2 pretest" exceeded the critical value of $F_{.95}$. This indicated that the difference between the means of the two pretest-achievement-level groups, "U" and "L", was significant at the .05 level. The mean of the "U" group was 28.74, and the mean of the "L" group was 19.05. Hence, the difference was in favor of the "U" group.

Analysis of Variance for the Three-Factor Factorial Design at the Ninth Grade Level on the Unit 2 Retention Test

An analysis of variance for the three-factor factorial design at the ninth grade level was performed on the retention data. The results of this analysis are summarized in Table 62 (Appendix C, p.153). The F-ratio obtained in the analysis of variance for the treatments factor did not exceed the critical value of $F_{.95}$.

The F-ratios for the interactions TA and AC:T did not exceed the critical values of $F_{.95}$. This was the same result that was obtained on the Unit 2 achievement data.

The obtained value of F for the classes factor exceeded the F_{95} value. This indicated that some of the class means (or totals) were significantly different at the .05 level. The Newman-Keuls test was again used to detect the specific pairs of class totals which differed. To aid in the present discussion, the ninth grade class totals on the Unit 2 retention test are arranged in increasing order of magnitude in Table 63 (Appendix C, p.).

The results of the Newman-Keuls test at the .05 level of significance showed that Classes 8 and 10 differed from Classes 13, 2, and 15, but Classes 8 and 10 did not differ from Class 1 or from each other. Since Classes 8 and 10 were both Method E classes, there was a trend toward the superiority of Method E, but as pointed out above the differences among treatment groups were not large enough to be considered statistically significant. The results also shows that Class 1 differed from Classes 13 and 2. The only two classes within the same treatment which differed were Class 1 and Class 2, both of which were Method D classes.

The only significant F-ratio besides that for the classes factor was that for the "achievement level on the Unit 2 pretest" factor. The obtained F-ratio for the latter exceeded the critical value of $F_{.95}$. Hence, the difference between the pretest-achievement-level group means on the retention test was significant at the .05 level. The mean of the "U" group was 27.43, and the mean of the "L" group was 17.96. The difference was therefore again in favor of the "U" group.

Analysis of Variance for the Four-Factor Factorial Design at the Eighth Grade Level on the Unit 2 Achievement Test

In the analysis of the four-factor factorial design at the eighth grade level "treatments" (T), "schools" (S), and "achievement level on the Unit 2 pretest" (A) were treated as fixed factors. "Classes" (C), treated as a random factor, was considered to be nested in "treatments" and "schools". The 24 subclasses which resulted from blocking on these factors were considered to have $\bar{n}_h$ observations each, where $\bar{n}_h$ was the harmonic mean of the actual number of students in each pretest-achievement-level subclass. That is, the "U" subclass of each class was considered to have $\bar{n}_h$ students and the "L" subclass of each class was considered to have $\bar{n}_h$ students. The value of $\bar{n}_h$ used in the eighth grade analysis was 7.17.

Table 64 (Appendix C, p.155) summarizes the analysis of variance for the four-factor factorial design at the eighth grade level on the Unit 2 achievement test. The obtained F-ratio for the treatments factor did not exceed the critical value of $F_{.95}$. This indicated that at the eighth grade level the differences among treatment means in achievement in Unit 2 were not significant at the .05 level.

Other factors whose F-ratios did not exceed the critical values were as follows: S(schools), TS (interaction of treatments and schools), TA (interaction of treatments and achievement level on the Unit 2 pretest), SA (interaction of schools and achievement level on the Unit 2 pretest), AC:TS (interaction of achievement level on the Unit 2 pretest and classes), and TSA (interaction of treatments, schools, and achievement level on the Unit 2 pretest).

The obtained F-ratio for the class factor exceeded the critical value of $F_{.95}$. This indicated that some of the differences among class means (or totals) were significant at the .05 level. The Newman-Keuls test was used to determine which pairs of class totals differed.

In Table 65 (Appendix C, p.156) the eighth grade class totals on the Unit 2 achievement test are arranged in increasing order of magnitude. Each class total was computed by finding the product of $\bar{n}_h$ and each subclass mean and then adding these two products.

The results of the Newman-Keuls test on the class totals at the .05 level of significance were as follows:Classes 14, 16, 12 and 17 differed from Class 9; and Classes 12 and 17 differed from Class 18.

Another result of the analysis of variance at the eighth grade level on the Unit 2 achievement test was that the obtained F-ratio for the "achievement level on the Unit 2 pretest" factor exceeded the critical value of $F_{.95}$. This indicated that the means of the two pretest level groups, "U" and "L", differed at the .05 level of significance. The mean of the "U" group was 28.03, and the mean of the "L" group was 21.29. Thus, the significant difference was in favor of the "U" group.

Analysis of Variance for the Four-Factor Factorial Design at the Eighth Grade Level on the Unit 2 Retention Test

An analysis of variance for the four-factor factorial design at the eighth grade level was performed on the Unit 2 retention data. Table 66 (Appendix C, p. 157) summarizes the results of this analysis. The obtained F-ratio for the treatments factor did not exceed the critical value of $F_{.95}$. This indicated that at the eighth grade level the differences among treatment means on the Unit 2 retention test were not significant at the .05 level.

Other factors whose F-ratios did not exceed the critical values were as follows: S(schools), TS (interaction of treatments and schools), TA (interaction of treatments and achievement level on the Unit 2 pretest), SA (interaction of schools and achievement level on the Unit 2 pretest), and TSA (interaction of treatments, schools, and achievement level on the pretest.)

As was the case on the Unit 2 achievement test, the F-ratio of the classes factor exceeded the critical value of $F_{.95}$. This indicated that some of the differences among class means (or totals) were significant at the .05 level. The Newman-Keuls procedure of determining which specific pairs of class totals differed was again applied. As in the preceding analyses, only the results of the Newman-Keuls test are reported here. Table 67 (Appendix C, p. 158) shows the eighth grade class totals on the Unit 2 retention test arranged in increasing order of magnitude.

The results of the Newman-Keuls test at the .05 level of significance may be summarized as follows: Classes 16, 6, 4, 7, 5, 14, 3, 12, and 17 differed from Class 9 and Class 17 also differed from Classes 18, 11, and 16 at the .05 level. There were several instances in which classes within the same treatment differed. Classes 7 and 12, which were Method E classes, differed from Class 9, a Method E class also. Class 17, a Method S class, differed from Classes 18 and 16, which were also Method S classes.

The F-ratio for the "achievement level on the Unit 2 pretest" factor exceeded the critical value of $F_{.95}$. Hence, the difference between the means of the two pretest-achievement-level groups, "U" and "L", was significant at the .05 level. The mean of the "U" group was 26.56, and the mean of the "L" group was 19.57. This indicated that the significant difference between means at the .05 level was in favor of the "U" group.

Another result of the analysis on the Unit 2 retention test was that the F-ratio of the factor AC:TS (interaction of achievement levels on the Unit 2 pretest and classes) exceeded the critical value of $F_{.95}$. That is, the interaction AC:TS was significant at the .05 level. This indicated that the difference between pretest-achievement-level subclass means on the Unit 2 retention test was not the same for every class. In a majority of the classes, the mean score of the "U" subclass was greater than the mean score of the "L" subclass. For example, in Class 16, the mean of the "U" subclass was 30.4 and the mean of the "L" subclass was 11.5. However, in Class 9, the mean of the "L" subclass (15.4) exceeded the mean of the "U" subclass (15.0)

Unit 3 Results

The results of Unit 3 are reported in the following manner: first, those analyses for which the F-values were non-significant at the .05 confidence level; second, those analyses for which the F-values in group mean comparisons were significant at the .05 level, and third, those analyses for which selected first-order interactions were significant at the .05 level.

Analyses for Which F-Values Were Non-Significant in Group Comparisons

When the criterion was achievement in Unit 3, there were no significant differences among the classification group means when the data were stratified by (1) student sex, (2) grade level, (3) treatment (method) within the ninth grade, (4) treatment within the subset of male students, (5) treatment within the subset of eighth grade boys, (6) treatment within the subset of ninth grade boys, (7) treatment within the subset of male students of female teachers, (8) treatment within the subset of female students of male teachers, and (9) treatment within the subset of male students of male teachers. Results of the analyses under stratification by treatment within the ninth grade may be inaccurate due to significant heterogeneity of regression at the .01 level. All other results under the stratifications listed above are based on the analysis of covariance with homogeneity of regression accepted at the .05 confidence level.

A summary of the preceeding analyses, together with adjusted comparison group means, are given in Table 68 (Appendix C, p.159). When the criterion was retention in Unit 3, there were no significant differences among classification group means when the data were stratified by (1) method, (2) student sex, (3) grade level, and (4) treatment within each of the following subsets: (a) eighth grade, (b) students of female teachers, (c) students of male teachers, (d) male students, (e) female students, (f) students of School 1, (g) students of School 2, (h) eighth grade girls, (i) eighth grade boys, (j) ninth grade boys, (k) female students of female teachers, (l) male students of female teachers, (m) female students of female teachers, and (n) male students of male teachers. For all of the preceeding analyses of Unit 3 retention data, heterogeneity of regression was non-significant. A summary of the preceeding analyses of Unit 3 retention data, together with adjusted group means, appears in Table 69 (Appendix C, p.).

Analyses of Unit 3 for Which F-Values Were Significant in Group Comparisons

For each stratification of the data the results of the analysis of covariance are reported first when the criterion was achievement in Unit 3 and second when the criterion was retention in Unit 3.

Stratification by Treatment (Method)

Table 70 (Appendix C, p. 161) presents the analysis of covariance of the Unit 3 achievement data when the subjects were grouped by method only. The analysis revealed significant differences at the .01 level among treatment means and no significant heterogeneity of regression. Table 71 (Appendix C, p 162) gives the results of the range tests. There was a significant difference at the .01 level between the means of the Method D group and the Method E group, a significant difference at the .05 level between the means of the Method S group and the Method D group, and no significant difference between the means of the Method E group and the Method S group. The adjusted means were $M_D = 17.38$, $M_E = 18.92$, and $M_I = 18.73$. As shown earlier, the analysis of covariance of the Unit 3 retention data when the subjects were grouped by method revealed significant differences among neither the treatment group means nor the regression equations.

Stratification by Teacher

Table 72 (Appendix C, p. 163) presents the analysis of covariance of the Unit 3 achievement data when the subjects were grouped by teacher. The analysis showed significant differences at the .01 level among teacher group means and no significant heterogeneity of regression. Table 73 (Appendix C, p. 164) summarizes the binary comparisons of teacher group means. Thus, at the .05 confidence level, group means significantly differed for Teacher 1 and Teacher 3, for Teacher 1 and Teacher 4, for Teacher 2 and Teacher 5, for Teacher 4 and Teacher 5, for Teacher 5 and Teacher 6, and at the .01 confidence level the group means significantly differed for Teacher 3 and Teacher 5, for for Teacher 3 and Teacher 7, and for Teacher 4 and Teacher 7.

Table 74 (Appendix C, p. 165) presents the analysis of covariance of the Unit 3 retention data when the subjects were grouped by teacher. Teacher group means differed significantly at the .05 level and homogeneity of regression held for the analysis. The range tests showed that a significant difference at the .05 level existed between the means of the following pairs of groups: Teacher 1 and Teacher 5; Teacher 1 and Teacher 6; Teacher 2 and Teacher 3; Teacher 2 and Teacher 5; and Teacher 2 and Teacher 7.

Stratification by Period of Day

Table 75 (Appendix C, p. 166) presents the analysis of covariance of the Unit 3 achievement data when the subjects were grouped by period of day. The results showed significant differences at the .05 level among period group means and no significant differences among regression equations. Four paired comparisons revealed significant differences. There was a significant difference between the group means of Period 1 and Period 3, of Period 2 and Period 4, of Period 3 and Period 4, and of Period 4 and Period 5.

Table 76 (Appendix C, p.167) presents the analysis of covariance of the Unit 3 retention data when the subjects were grouped by the class period in which they took general mathematics. The analysis showed significant differences at the .01 level among period group means and no significant heterogeneity of regression. The results of the range tests revealed a significant difference at the .01 level between the means of Period 4 and Period 5 only. However, at the .05 level, there were significant differences between the means of Period 1 and Period 5, of Period 2 and Period 5, of Period 3 and Period 5, O period 4 and Period 6, and of Period 5 and Period 6.

Stratification by School

Table 77 (Appendix C, p.169) presents the analysis of covariance of the Unit 3 achievement data when the subjects were grouped by school. The analysis indicated a significant difference between school achievement means at the .01 level. Thus, the adjusted mean of School 1 (15.93)was inferior to the adjusted mean of School 2 (19.15)

Table 78 (Appendix C, p. 170) presents the analysis of covariance of the Unit 3 retention data when the subjects were grouped by school. The analysis indicated a significant difference between school retention means at the .01 level. These retention means were consistent with those for achievement since the adjusted mean for School 1 was 15.93 and the adjusted mean for School 2 was 17.04.

Stratification by Method Within the Eighth Grade.

Table 79 (Appendix C, p.171) presents the analysis of covariance of the Unit 3 achievement data when the eighth grade subjects were grouped by method of instruction. There were significant differences at the .05 level among treatment means and no significant heterogeneity of regression. There was a significant difference between Method D and Method E and between Method D and Method S but no significant difference between Method E and Method S. As was shown previously, there were significant differences at the .05 level among neither treatment means nor regression equations when the criterion was retention.

Stratification by Method Within the Ninth Grade

Table 80 (Appendix C, p. 172) presents the analysis of covariance of the Unit 3 retention data when the ninth grade subjects were grouped by method. There were significant differences among the retention means at the .01 level with no reason to suspect homogeneity of regression. Further analyses revealed a significant difference between Method D and Method E at the .01 level, a significant difference between Method E and Method S at the .05 level. These results and the adjusted retention means are listed in Table 81 (Appendix C, p. 173).

Stratification by Method Within the Female Teacher Subgroup.

Table 82 (Appendix C, p. 175) presents the analysis of covariance of the Unit 3 achievement data when the subjects of the female instructors were grouped by method. The analysis showed significant differences at the .01 level among treatment means and no significant differences among regression equations. Table 83 (Appendix C, p. 176) summarizes the results of the range tests. These results showed Method E superior to Method D at the .01 level, but Method S superior to Method D at the .05 level, and Method E superior to Method S at the .05 level.

The analysis of covariance of the Unit 3 retention data when the subjects of the female instructors were grouped by method was presented earlier. In contrast to the striking differences among achievement means, no significant differences existed among retention means.

Stratification by Method Within the Male Teacher Subgroup.

Table 84 (Appendix C, p. 177) presents the analysis of covariance of the Unit 3 achievement data when the subjects of male instructors were grouped by method. The analysis indicated significant differences at the .05 level among achievement means. However, heterogeneity of regression was significant at the same confidence level. Also, as previously reported there were significant differences at the .05 level among neither retention means nor regression equations when the criterion was retention.

Stratification by Method Within the Female Student Subgroup

Table 85 (Appendix C, p. 178) presents the analysis of covariance of the Unit 3 achievement data when the female subjects were grouped by method. There were significant differences at the .05 level among treatment means. Further analyses showed Method E superior to Method D and Method S superior to Method D at the .05 level and no significant difference between Methods E and S.

Stratification by Method Within School 1.

Table 86 (Appendix C, p. 179) presents the analysis of covariance of the Unit 3 achievement data in School 1 when the subjects were grouped by method. Although the analysis indicated that treatment means differ at the .01 level, heterogeneity of regression existed at the same level. The adjusted means of $M_D = 17.03$, $M_E = 18.61$, and $M_S = 18.71$ suggested significant differences, but the technique by which they were derived was probably inappropriate for this classification of the data.

Stratification by Method Within School 2.

Table 87 (Appendix C, p. 180) presents the analysis of covariance of the Unit 3 achievement data when the subjects of School 2 were grouped by method. There were significant differences at the .05 level among the achievement means at School 2 and no significant differences among regression equations. Results showed that Method D and Method E group means differed, but neither the Method D nor the Method E group mean differed from the Method S group mean.

Stratification by Method Within the Eighth Grade Female Student Subgroup.

Table 88 (Appendix C, p. 181) presents the analysis of covariance of the Unit 3 achievement data when the eighth grade girls were grouped by method. The analysis showed significant differences among the treatment means at the .05 level. Results of the range tests, showed that the mean of the Method E group was significantly greater than the mean of the Method D group and that there was no significant difference between the means of either the Method D and Method S groups or the Method E and Method S groups.

Stratification by Method Within the Ninth Grade Female Student Subgroup.

Table 89 (Appendix C, p. 182) presents the analysis of covariance of the Unit 3 achievement data when the ninth grade girls were grouped by method. The results indicated that there were significant differences among the adjusted means. However, heterogeneity of regression was significant at the .01 level indicating that the results were unreliable, perhaps inaccurate.

Table 90 (Appendix C, p. 183) presents the analysis of covariance of the Unit 3 retention data when the ninth grade girls were grouped by method. The analysis showed significant differences at the .05 level among treatment means and no significant heterogeneity of regression. When subjected to further analysis, a significant difference was found to exist between the means of the Method D group and the Method E group only.

Stratification by Method Within the Female Students of Female Teachers Subgroup.

Table 91 (Appendix C, p. 184) presents the analysis of covariance of the Unit 3 achievement data when the female students of female instructors were grouped by method of instruction. The analysis showed significant differences among the treatment means at the .01 level and no significant heterogeneity of regression. The results of the multiple range tests indicated that the means of the Method D group, and the Method E group differed at the .01 level, the means of the Method E group and the Method S group differed at the .05 level, and the means of the Method D group and the Method S group did not significantly differ.

Analyses for Which Selected First-Order Interactions were Significant

For achievement in Unit 3, there were two significant, first-order interactions. The first significant first-order interaction was between method and teacher at the .01 level and is presented in Table 92 (Appendix C, p. 185). Thus, teacher and method did not exercise separate and independent effects upon achievement in Unit 3 by the eighth and ninth grade general mathematics students. The interaction is shown diagramatically in Figure 8 (Appendix D, p. 201). Notice that the means of the Method D group and the Method E group exchanged order relation from Teacher 3 to Teacher 6, and the means of the Method E group and the Method S group exchanged order relations from Teacher 1 to Teacher 7.

Table 93, (Appendix C, p. 186) presents the two-way analysis of variance of the Unit 3 achievement data with factors teacher and mental ability level. The analysis revealed a significant interaction at the .05 level between the two factors. Thus, relative order of achievement in Unit 3 by the different mental ability groups varied from teacher to teacher as illustrated in Figure 9 (Appendix D, p. 202). Notice in Figure 9 that the ability groups retain their relative positions across teachers but do not vary uniformly. Notice that the average and high ability group means have similar variations, while the low ability group tends in opposite directions for Teachers 1, 2, 3, and 7.

Table 94 (Appendix C, p. 187) presents the two-way analysis of variance of the Unit 3 retention data when the subjects were grouped by method and teacher. The analysis indicated a significant interaction, at the .01 level, between method and teacher. Thus different teachers had different effects upon retention by the various ability groups. This interaction is illustrated graphically in Figure 10 (Appendix D, p. 203). The results showed that the order was reversed for the adjusted means (a) of the Method D and Method S groups from Teacher 2 to Teacher 4, (b) of the Method D and Method E groups from Teacher 3 to Teacher 6, and (c) of the Method E and Method S groups from Teacher 1 to Teacher 7.

Table 95 (Appendix C, p. 188) presents the analysis of variance of the Unit 3 retention data when the subjects were grouped by method and mental ability level. The analysis showed a significant interaction between the two factors at the .05 level. This interaction effect is illustrated in Figure 11 (Appendix D, p. 204). The adjusted means indicated (a) that below average IQ subjects retained Unit 3 best under Method S and poorest under Method E, (b) that the average IQ subjects retained Unit 3 equally well under Methods D and E but slightly better under Method S, and (c) that the above average IQ subjects retained best under Method E and poorest under Method D. Thus the data indicated that the effectiveness of Method E, relative to Methods D and S, on retention of Unit 3 increased directly with intelligence.

Table 96 (Appendix C, p. 190) presents the analysis of variance of the Unit 3 retention data when the subjects were grouped by teacher and prior general mathematical achievement level. This analysis revealed a significant interaction (at the .01 level) between the two factors. Thus, the interaction of the factors produced effects which were not separate and independent. Figure 12 (Appendix D, p. 205) indicates that the effect of Teacher 5 on retention of Unit 3 was greater on the low achievement group than that of the other teachers. The relative effectiveness of each of the other teachers on the various achievement level groups was consistent.

Table 97 (Appendix C, p. 190) presents the analysis of variance of the Unit 3 retention data when the subjects were grouped by teacher and mental ability level. This analysis revealed a significant interaction at the .01 level between the factors teacher and mental ability. Thus, the factors did not exercise separate and independent effects upon retention of Unit 3,as shown in Figure 13, (Appendix D, p.). The effect of Teacher 5 on the retention of the low IQ group was inconsistent with the effects of the other teachers on groups with the same mental ability level.

Analysis of the Student and Teacher Attitude Data

Table 98 (Appendix C, p.191) lists the teacher attitude pretest scores (X_{pre}), attitude posttest scores (X_{post}), the difference scores $(D)_1$, the squared difference scores (D^2), and a t-test of the null hypothesis that there is no change in teacher attitude (the population difference is zero). Since $|t|$ (the absolute value of t) is less than $t_{.95} = 1.94$, the null hypothesis is accepted.

The t-test was also applied to the student data collected by Dutton's "Attitude Toward Mathematics Scale". The analysis included only those students for whom all other data were available (N = 294.) For each student, a "gain" score was computed by subtracting the score made when the scale was used as a pretest from the score made when the scale was used as a posttest. The 294 students were partitioned into six groups (8th grade -Method D, 8th grade - Method E, 8th grade-Method S, 9th grade-Method D, 9th grade-Method E, 9th grade-Method S), and the hypothesis that mean gain score was zero was tested for each of the six groups. The results of the t-tests are summarized in Table 99 (Appendix C, p. 192).

The obtained value of t was non-significant in each case. Hence, for each group named above, the data supported the hypothesis that the mean gain score was zero.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The conclusions reached from the results of statistically testing the research hypotheses, when stated in the null form, constitutes the first major section of this chapter. Recommendations for use of, and further research related to, the present study follow the conclusions.

Conclusions

The conclusions of the study are presented first in order by unit and then for the entire study.

Unit 1

The results of the study suggested that method of instruction had no differential effects upon achievement for the following groups: (a) School 2; (b) male students; (c) eighth grade male students; and (d) male students of female teachers. Also,method of instruction had no differential effects upon retention for the following groups: (a) male students; (b) eighth grade male students ;(c) ninth grade male students; (d) male students of female teachers; and (e) male students of male teachers. The variables (a) sex of student, (b) grade level, and (c) period of day did not effect achievement in Unit 1 and the variables (a) sex of student and (b) grade level did not effect retention in Unit 1.

The results of the study revealed differential effects of the treatments (methods) upon achievement for several stratifications of the data. The most common results of the effects showed Method D achievement inferior to Method E and Method S achievement but no difference in achievement of Method E and Method S. These results were found by the following stratifications of the data: (a) treatment; (b) treatment within the students of female teachers; (c) treatment within the eighth grade female students; and (d) treatment within the female students of female teachers. Treatment also exerted differential effects upon retention for many stratifications of the data. Again, the most common results showed Method D retention inferior to Method E and Method S retention but no real difference between Method E and Method S retention. These results were found by the following stratifications of the data: (a) treatment; (b) treatment within the eighth grade; (c) treatment within the ninth grade; (d) treatment within the female teachers; (e) treatment within the students of male teachers; (f) treatment within the female students; (g) treatment within School 1; (h) treatment within eighth grade girls; (i) treatment within ninth grade girls; (j) treatment within female students of female teachers; and (k) treatment within female students of male teachers. Thus method of instruction had a differential effect upon both achievement and retention by the female subjects but not the male subjects. This conclusion was drawn on the basis of those analyses for which homogeneity of regression was accepted.

Student sex and grade level exerted differential effects upon neither achievement nor retention in Unit 1. The variables teacher and school exercised differential effects upon both achievement and retention, and period of day exercised differential effects upon retention. However, these variables were confounded in the design of the study and thus no conclusions can be drawn concerning the effectiveness of these variables. The results of the analysis for which the basic assumption of homogeneity of regression was suspect suggested the same general results as stated above.

The covariates were found to be good predictors, when pooled and applied collectively, of student achievement and retention. When classified and inserted as factors in a two-way analysis of variance, prior general mathematical achievement was found to be the one best predictor of achievement and retention. Stratification by each of the classification variables generally yielded the expected results. However, there were instances of a significant first-order interaction between variables.

Unit 2

Included among the hypotheses of Unit 2 that were not supported by the data were the hypotheses designed to test the relative effectiveness of the three teaching methods (Method D, Method E, and Method S), upon achievement and retention in Unit 2 by (a) the composite population, (b) the population of ninth grade students, and (c) the population of eighth grade students. Specifically, the two hypotheses stated for each of the three populations were: (1) that the three treatments (teaching methods) have a differential effect upon achievement in the selected content, and (2) that the three treatments have a differential effect upon retention in the selected content. The two remaining hypotheses which were not supported by the data pertained to the population of eighth grade students and stated that the two schools (School 1 and School 2) have a differential effect on achievement and/or retention and (b) achievement and/or retention of students who score at or above the Unit 2 pretest median differs from the achievement and/or retention, respectively, of students who score below the Unit 2 pretest median.

Thus, in Unit 2, neither of the three teaching methods (Method D, Method E, and Method S) was found superior to one or more of the other two methods. Also, students who scored at or above the 50th percentile on the Unit 2 pretest scored significantly higher on both the achievement and retention tests of Unit 2 than the students who scored below the 50th percentile on the Unit 2 pretest.

Unit 3

The results of the study suggested that method of instruction had no differential effects upon achievement for the following groups: (a) male students; (b) eighth grade male students; (c) ninth grade male students ;(d) male students of female teachers; (e) female students of male teachers, and (f) male students of male teachers. Method of instruction had differential effects upon achievement for the following groups: (a) total sample; (b) eighth grade subjects; (c) subjects of female teachers; (d) female students; (e) School 2 students; (f) eighth grade female students; and (g) female students of female teachers. Thus method had differential effects upon female student achievement in Unit 3 but not upon male student achievement. The results generally showed Method D inferior to each of Method E and Method S, Methods E and S about equally effective, and Method E occasionally superior on achievement to Method S. These results indicated that achievement in Unit 3 was generally superior for the female subjects when a moderate degree of guidance was rendered. Achievement was poorest for the female subjects when a minimum degree of guidance was rendered. And, degree of guidance was immaterial relative to achievement by the male subjects of the study.

Treatment produced differential effects upon retention for practically no groups. For the ninth grade subjects method effects on retention were detected, but these effects were found to apply only to the female students in the ninth grade. Method D was slightly inferior to Method E, but not to Method S for the ninth grade female subjects. For no group of male subjects were differential effects produced by method of instruction.

The variables student sex and grade level exerted differential effects upon neither achievement nor retention in Unit 3. The variables teacher, period of day, and school each exerted differential effects on both achievement and retention. No conclusions can be drawn from the effects of these three variables since the variables were confounded in the design of the study.

The covariates were effective predictors, when pooled and applied collectively, of both student achievement and retention. When each covariate was classified into three levels and treated as a factor in the two-way analyses of variance, prior general mathematical achievement was isolated as the one best predictor of both achievement and retention. Stratification of the data by each of the classification variables generally yielded expected results, but there were instances of first-order interactions.

General Conclusions

Based on the results of all three units, neither of Methods D, E, and S is superior or inferior, relative to each other, in teaching the selected content of this study to male eighth and/or ninth grade general mathematics students when the criterion is achievement or retention in the selected content. (Retention here is measured by an achievement test delayed about five weeks after the immediate posttest.) The results of the study were not so clear cut for the female students. The results of Units 1 and 3 would suggest the conclusion that Method D is slightly inferior to each of the other two methods when the criterion is female student achievement. Unit 2 results indicate no differences among the methods, not supporting the results of Units 1 and 3. Eighth grade female student retention was affected by method of instruction only in Unit 1. These results suggested that after a period of adjustment eighth grade female students retention in the selected content is not significantly affected by method of instruction. Ninth grade female student retention was not affected by method only in Unit 2. On the other hand, when several other subsets of students in the study involved ninth grade girls (such as the entire sample,the female students, the female students of female teachers, and the female student of male teachers), method had no differential effect upon retention. Thus, method of instruction had a very small differential effect upon retention in the selected content of Units 2 and 3.

In each unit teacher fidelity to his assigned teaching methods was somewhat less than desired. However, the rating scales were rather crude and fidelity did seem to increase in successive units.

There were no significant changes in attitude toward mathematics for either teachers or students. Thus method of instruction had no differential effect upon either teacher or student attitude toward mathematics.

Recommendations

For practical use or re-evaluation purposes, the following implications are suggested:

1. the results of this study do not support the theory that less-directed methods and more-directed methods (to the extent that Methods D, E, and S can be considered characteristic of more directed and less directed methods in general) are differentially effective with general mathematics students when effectiveness is measured in terms of paper-and-pencil-achievement tests delayed approximately six weeks to measure retention.

(2.) The results of this study do not support the theory that more-directed methods are inherently superior to less-directed methods in producing achievement on immediate posttest by male students (inconclusive for female students).

(3.) The results of this study do not support the theory that less-directed methods are inherently superior to more-directed methods in producing achievement on delayed posttests.

(4.) The results of this study support the theory that the more able general mathematics students (in terms of pretest performance), as compared with the less able students, demonstrate superior achievement on paper-and-pencil-achievement posttests regardless of whether they are taught by more directed methods or less-directed methods.

(5.) The teaching methods described in this study provide the classroom teacher with at least three different approaches to teaching general mathematics. Perhaps one of these methods, or a combination of them, could be used as a model in planning for instruction in other content areas. The teacher may find that one approach is more suitable for a certain body of content than another. Or, one approach may seem to suit the personality of a particular class more so than another. At any rate, the models are available to classroom teachers, and each teacher may use his own judgment in applying the models; and

(6.) If the classroom teacher uses the instructional materials and measuring instruments employed in this study, he can probably expect the students who score higher on the Pretest to maintain their superiority on the two posttests. The Pretest might serve as a basis for intraclass grouping and would thus allow the teacher to tailor his instruction to the needs of the different Pretest-achievement-level groups.

The recommendations presented below should be accepted only under the limitations of the study and applied only to related learning situations where the mathematics, the teachers, the students, and the environment have been closely paralleled.

(1) The eighth or ninth grade general mathematics teacher should feel free to use any one of Methods D, E, and S without fear of significant deprivation of student retention due to method of instruction, provided the instructional period is for approximately six weeks or more.

(2.) For a short period of instruction (up to 3 weeks), Method D should not be substituted for a conventional method on a group composed of female, or predominately female, subjects. However, Methods E and S were equally effective for groups of this type.

(3.) For a group of male, or predominately male, eighth or ninth grade general mathematics students the choice of method of instruction--Method D, Method E, or Method S--should be determined by factors other than effects of the method on achievement or retention in the selected content regardless of the length of the instructional period.

(4.) Only achievement and retention in the selected content were evaluated in this study. Similar studies should be conducted with criterion variables such as critical-thinking, mathematical reasoning, transfer of learning, and learning "how to discover."

(5.) More stringent techniques than those of this study should be employed to ensure teacher fidelity to assigned teaching methods. The results suggest that the teachers involved in a methods study should be instructed for more than two sessions before participating in an instructional program involving two or more methods. It is also recommended that they actually teach by the methods under the supervision of the researcher prior to the instructional period of the experimental study. If a student is to rate classroom procedures, it is recommended that he be instructed in discriminating among the various types of classroom behavior and given actual rating practice under supervision.

(6) Although the reliability coefficients of the criterion instruments were satisfactory, they were somewhat less than desired in Units 1 and 3. If the tests are to be used again, it is suggested that they be refined and more items added in order to elevate their reliabilities. Also, the criterion measures used in this study should be modified to include items which would indicate how a student arrived at his answers and whether a student solved a problem by applying a mathematical principle.

(7.) The teaching methods in this study were compared on the basis of student achievement and retention in the selected content. A study using the selected content, but using different criteria for comparison, such as "increase in quantity or quality of creative and independent thinking," "attainment of intrinsic rewards," or "ability to apply the mathematical principles in laboratory-type problems," is in order.

(8.) The teaching methods developed for use in this study should be compared at other grade levels using students of varying maturity in mathematics.

(9.) Since classes were nested in treatments in this study, the design did not allow an examination of possible interactions between classes and treatments. Future studies should be designed to allow investigation of whether treatment effects differ from class to class.

(10.) At the completion of this study, several of the participating teachers expressed the belief that "some kinds of student personalities" seem to react more favorably to a particular method than "other kinds of student personalities." A study of the kinds of student personalities which react favorably and unfavorably to the teaching methods is recommended.

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APPENDIX A

LETTERS

C O P Y

May 3, 1967

James D. Gates, Executive Secretary
National Council of Teachers of Mathematics
2101 Sixteenth Street, N.W.
Washington, D.C.

Dear Mr. Gates:

We are planning to conduct an in-service institute for teachers of "general mathematics" in the Bibb County (Macon), Georgia, area during 1967-68, and we wish to explore the possibility of using modifications of Experiences in Mathematical Discovery with the students (approximately 500) of the participating teachers.

The study we envision now is to evaluate the effectiveness of several teaching procedures and how these procedures interact with concurrent training of teachers. After reviewing several sets of instructional materials, we feel that the EMD series would provide the best content basis for the general mathematics students with whom we would be working.

We are requesting permission to construct and use modified versions of the first five pamphlets of the EMD series. The modifications would be constructed so as to facilitate a comparison of four different teaching methods based on increased amounts of guidance in the discovery process. The modifications would consist of both duplicating and paraphrasing the content in the pamphlets.

I have recently discussed this proposal with Dr. Donovan Johnson at the Las Vegas Convention. He suggested that I officially request permission to use these materials as described above.

Sincerely,

//s//

Len Pikaart, Chairman
Mathematics Education

//s//

William D. McKillip
Assistant Professor of Mathematics
Education

LP:WDM:hk
cc: Dr. Donovan Johnson

C O P Y

May 22, 1967

Dr. Len Pikaart
The University of Georgia
College of Education
Athens, Georgia

Dear Dr. Pikaart:

At a meeting of the Executive Commiteee last week, permission was granted for the procedures outlined in your letter of May 3 involving the reproduction of portions of our publication entitled "Experiences in Mathematical Discovery". We would appreciate it if you could send copies of the results of this project as I am sure our Board of Directors and some of our committees would be interested in them.

Sincerely yours,

//s//

James D. Gates

JDG/haw
cc: Dr. Donovan A. Johnson

APPENDIX B

LESSON 8

(Sample Lessons)

LESSON 8 (METHOD D)

In this lesson we are going to study a special type of polygon called a triangle. We have learned that a triangle is a 3 sided polygon. We are going to compare the measures of the angles of triangles. As you do this lesson, try to find a relationship between the measures of the angles of triangles.

You will need your straight edge and protractor to do this lesson. If you have forgotten how to use a protractor, be sure to ask your teacher for help. Success in this lesson depends on being able to measure angles with a protractor. So, be very careful when measuring angles.

Discussion 8

Make all measures to the nearest degree.

1. Look at the triangle in Fig. 8.1.

   a. Measure ∠A. m∠A = ________.

   b. Measure ∠B. m∠B = ________.

   c. Measure ∠C. m∠C = ________.

   d. Add your answers to parts

   a, b, and c.

   m∠A + m∠B + m∠C = ________.

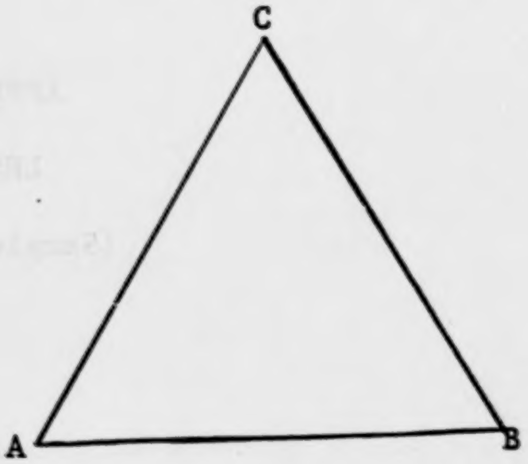


Fig. 8.1

2. Look at the triangle in Fig. 8.2

   a. Measure ∠D. m∠D = ________ .

   b. Measure ∠E. m∠E = ________ .

   c. Measure ∠F. m∠F = ________ .

   d. Add your answers to parts

   a, b, and c.

   m∠D + m∠E + m∠F = _______.

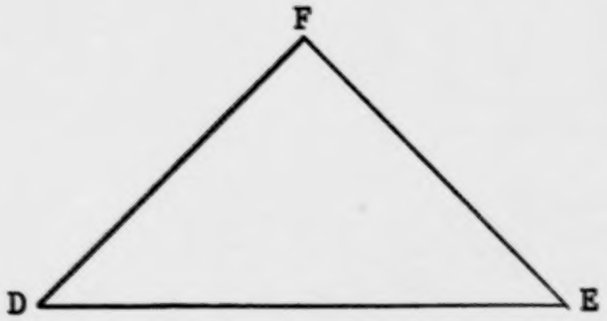


Fig. 8.2

3. Look at the triangle in Fig. 8.3.

   a. m∠G = ________.

   b. m∠H = ________.

   c. m∠I = ________.

   d. m∠G + m∠H + m∠I = ________.

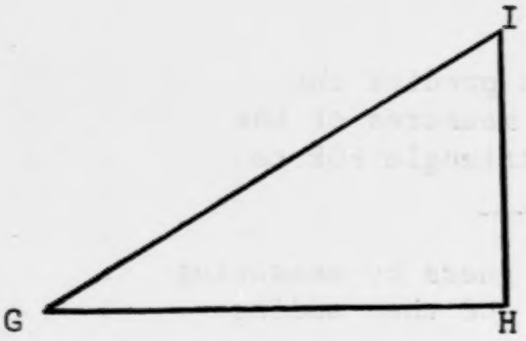


Fig. 8.3

4. Look at Fig. 8.2.

   a. m∠P = ________.

   b. m∠Q = ________.

   c. m∠R = ________.

   d. m∠P + m∠Q + m∠R = ________.

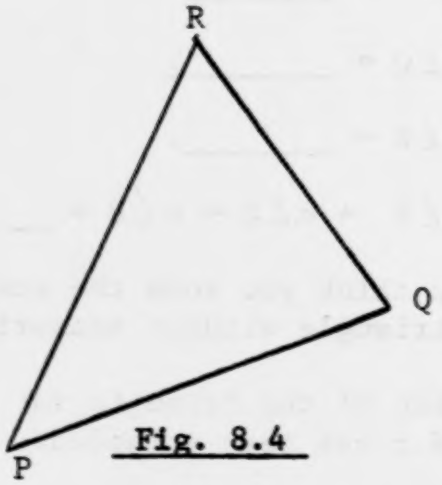


Fig. 8.4

5. Fill in the table below from numbers 1-4 of Discussion 8.

Triangle	Measures of angles			Sum of measures of angles
Fig. 8.1				
Fig. 8.2				
Fig. 8.3	30°	60°	90°	
Fig. 8.4				

Do you see a pattern in the "sum of measures of angles" column of the table? (**Note:** There may seem to be a pattern except for the sum being different by a few degrees. This is probably due to errors in measurement. Overlook small differences and try to find the pattern.)

6. What do you predict the sum of the measures of the angles of triangle PQR to be? ______

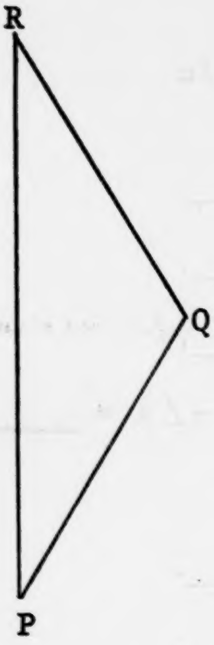
Check your guess by measuring each angle and then adding them.

a. m∠P = ______.

b. m∠Q = ______.

c. m∠R = ______.

d. m∠P + m∠Q + m∠R = ______.



7. Do you think you know the sum of the measures of the angles of a triangle without measuring the angles? ______

8. One side of the triangle in Fig. 8.5 has been extended.

a. Are ∠x and ∠y a linear pair of angles? ______

b. We know that m∠x + m∠y = 180° .

c. We also know m∠r + m∠s + m∠x = 180° .

d. So, if m∠x = 80°, m∠y = 100° .

And with m∠x = 80° m∠r + m∠s = 100° .

e. Suppose m∠x = 90°

Then m∠y = ______.

And, m∠r + m∠s = ______.

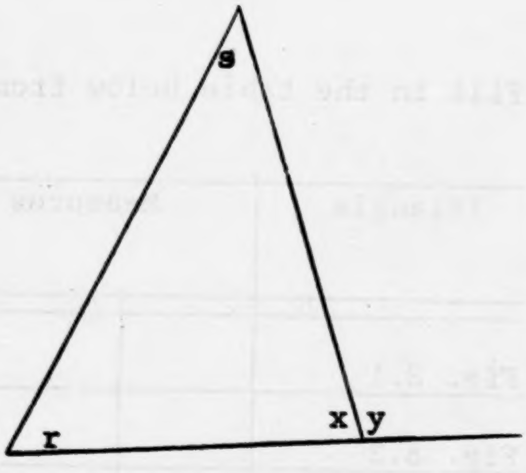


Fig. 8.5

9. A side of Fig. 8.6 has been extended to form ∠d.

a. Are ∠d and ∠b a linear pair? ______.

So, m∠b + m∠d = ______.

b. We know that m∠a + m∠b + m∠c = ______.

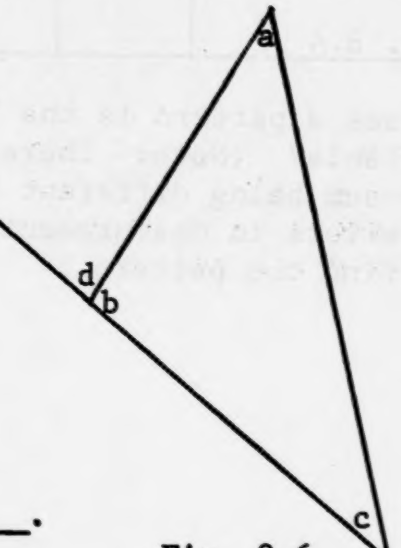


Fig. 8.6

c. If m∠b = 100°, then m∠d = ________.

Also, if m∠b = 100°, then

m∠a + m∠c = ________.

d. If m∠b = 10°, then m∠d = ________.

Also, if m∠b = 10°, then

m∠a + m∠c = ________.

10. Use your protractor on Fig. 8.7 to complete the statements below.

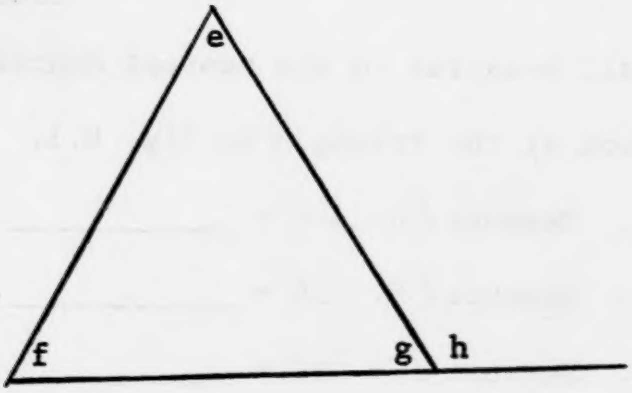


Fig. 8.7

a. m∠f = ________.

b. m∠e = ________.

c. Add your answers to parts a and b.

m∠f + m∠e = ________.

d. m∠h = ________.

e. Compare your answers to parts c and d.

(Errors in measurement may cause a difference of a few degrees. With this in mind you should be able to see a relationship.

11. Given any triangle like the triangle in Fig. 8.8, do you think there is a relationship between angle u and angles r and s? _____

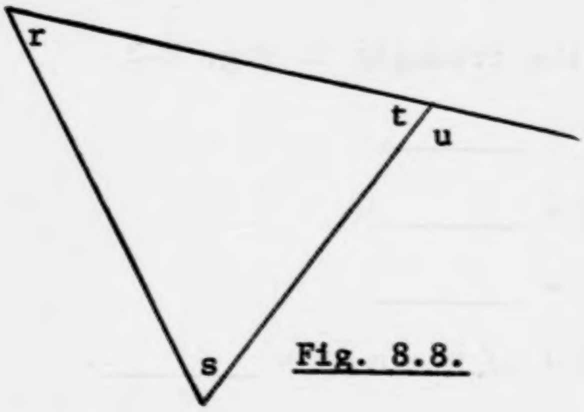


Fig. 8.8.

LESSON 8 (METHOD E)

In this lesson, we are going to study a special type of polygon called a triangle. We have learned that a triangle is a three-sided polygon. We are going to compare the measures of the angles of triangles. As you do this lesson, try to find a relationship between the measures of the angles of triangles.

You will need your straight edge and protractor to do this lesson. If you have forgotten how to use a protractor, be sure to ask your teacher for help. Success in this lesson depends on being able to measure angles with a protractor. So, be very careful when measuring angles.

Discussion 8

Make all measures to the nearest degree.

1. Look at the triangle in Fig. 8.1.

- **a. Measure ∠A. ∠A = ____________.**
- **b. Measure ∠B. ∠B = ____________.**
- **c. Measure ∠C. ∠C = ____________.**
- **d. Add your answers to parts a, b, and c.**
  m∠A + m∠B + m∠C = _________.

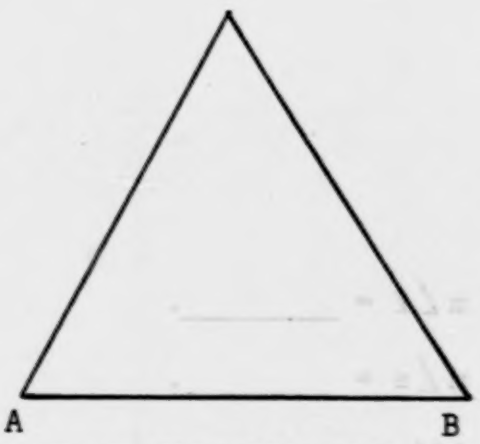


Fig. 8.1

2. Look at the triangle in Fig. 8.2.

- **a. Measure ∠D. m∠D = ________.**
- **b. Measure ∠E. m∠E = ________.**
- **c. Measure ∠F. m∠F = ________.**
- **d. Add your answers to parts a, b,**
  and c. m∠D + m∠E + m∠F = ________.

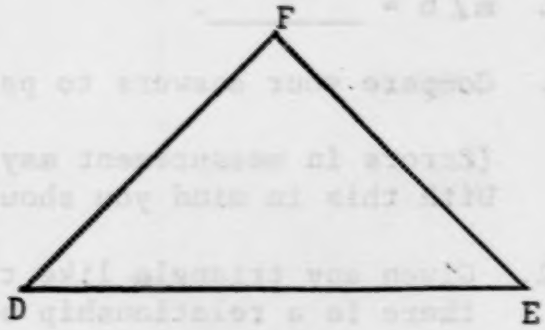


Fig. 8.2

3. Look at the triangle in Fig. 8.3.

- **a. m∠G = ________.**
- **b. m∠H = ________.**
- **c. m∠I = ________.**
- **d. m∠G + m∠H + m∠I = ________.**

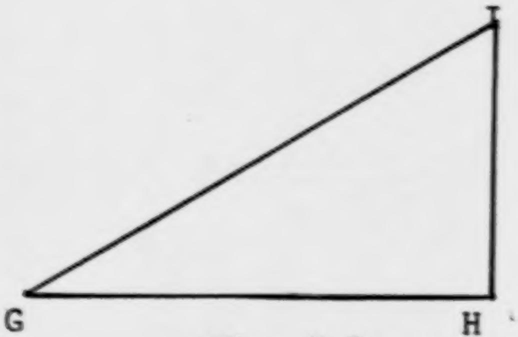


Fig. 8.3

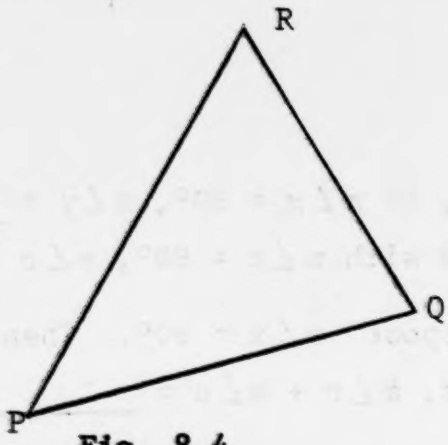


Fig. 8.4

4. Look at Fig. 8.4

   a. m∠P = ________.

   b. m∠Q = ________.

   c. m∠R = ________.

   d. m∠P + m∠Q + m∠R = ________.

5. Fill in the table below from numbers 1-4 of Discussion 8.

Triangle	Measures of angles			Sum of measures of angles
Fig. 8.1				
Fig. 8.2				
Fig. 8.3	30°	60°	90°	
Fig. 8.4				

Do you see a pattern in the "sum of measures of angles" column of the table? (Note: There may seem to be a pattern except for the sum being different by a few degrees. This is probably due to errors in measurement. Overlook small differences and try to find the pattern.)

6. What do you predict that the sum of the measures of the angles of triangle PQR will be? ________

   Check your guess by measuring each angle and then adding them.

   a. m∠P = ________.

   b. m∠Q = ________.

   c. m∠R = ________.

   d. m∠P + m∠Q + m∠R = ________.

7. Notice that the sum of the measures of the angles of any triangle is 180 degrees.

8. One side of the triangle in Fig. 8.5 has been extended.

   a. Are ∠x and ∠y a linear pair of angles? ________

   b. We know that m∠x + m∠y = 180°.

   c. We also know m∠r + m∠s + m∠x = 180°.

d. So, if m∠x = 80°, m∠y = ______.
And with m∠x = 80°, m∠r + m∠s = 100°

e. Suppose m∠x = 90°. Then m∠y = ______.
And, m∠r + m∠s = ______.

9. A side of Fig. 8.6 has been extended to form ∠d.

a. Are ∠d and ∠b a linear pair? ______
So, m∠b + m∠d = ______

b. We know that m∠a + m∠b + m∠c = ______.

c. If m∠b = 100°, then m∠d + ______.

Also, if m∠b = 100°, then

m∠a + m∠c = ______.

d. If m∠b = 10°, then m∠d = ______.

Also, if m∠b = 10°, then

m∠a + m∠c = ______.

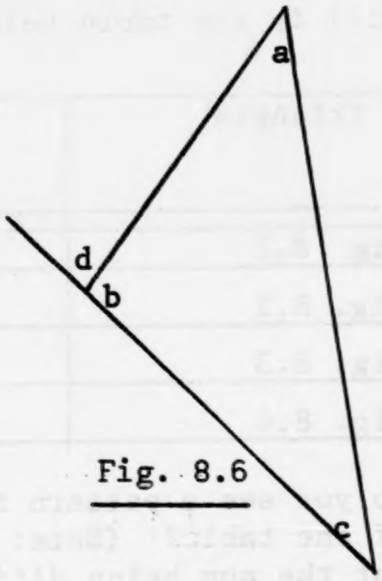


Fig. 8.6

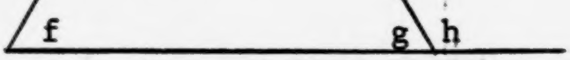
10. Use a protractor on Fig. 8.7 to complete the statements below.

a. m∠f = ______.

b. m∠e = ______.

c. Add your answers to parts a and b. m∠f + m∠e = ______.

d. m∠h = ______



e. Compare your answers to parts c and d.

Fig. 8.7

(Errors in measurement may cause a difference of a few degrees. With this in mind you should be able to see a relationship.)

11. Notice that in a triangle, like the one in Fig. 8.8, $m\angle r + m\angle s = m\angle u$.

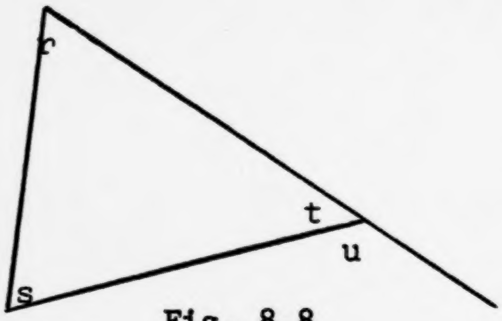
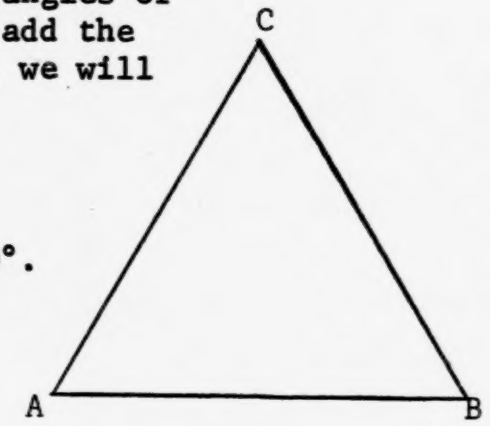


Fig. 8.8

LESSON 8 (METHOD S)

In this lesson we are going to study a special type of polygon called a triangle. We have learned that a triangel is a three-sided polygon. A triangle has many interesting properties. We will look at two of the properties in this lesson.

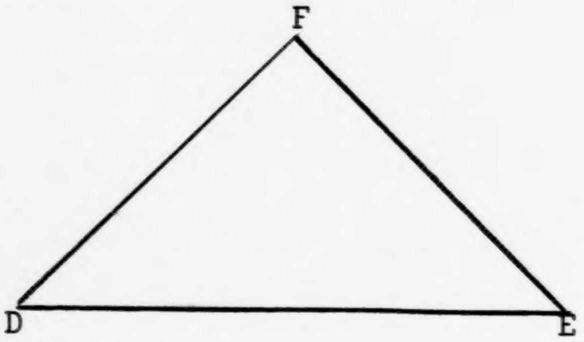
Property 1. The sum of the measures of the angles of any triangle is 180 degrees. That is, if we add the measures of the three angles of any triangle, we will always get 180°.



Example 1. Triangle ABC has angles that each measures 60°. So the sum of the measures if 60° + 60° + 60° or 180°.

Measure one of the angles of triangle ABC with your protractor.
What measure do you get? __________?

Problem 1. Measure the angles of triangle DEF. Find the sum of the measures of the angles.



a. $m\angle D =$ ________.

b. $m\angle E =$ ________.

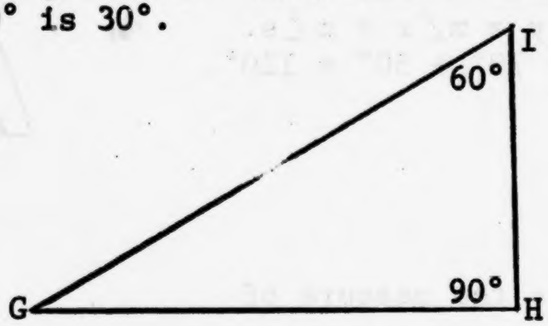
c. $m\angle F =$ ________.

d. $m\angle D + m\angle E + m\angle F =$ ________.

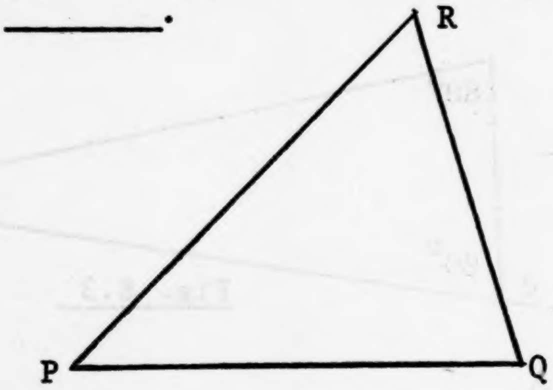
(Note: Your sum of the measures of the angles may not be exactly 180°. Measuring errors can cause the sum to be a few degrees off. So, overlook small differences.)

Example 2. Two of the angles of triangle GHI have measures of 90° and 60°. What is the measure of the third angle? ____

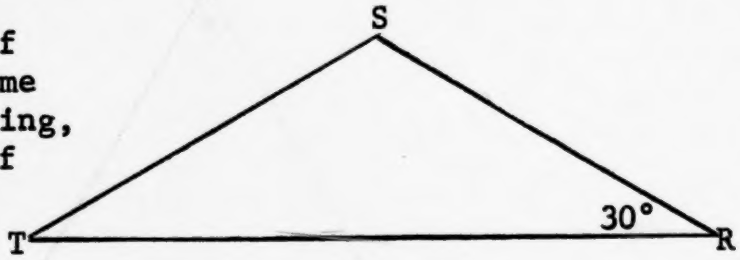
Since the sum of the measures of the angles if 180°, we subtract $m\angle H + m\angle I$ from 180° to get $m\angle G$. Now, $m\angle H + m\angle I = 90° + 60° = 150°$. And $180° - 150°$ is 30°.
So, $m\angle G = 30°$.



Problem 2. Measure angles P and Q of triangle PQR. Without measuring $\angle R$, determine what its measure is.
$m\angle R =$ ________.



Problem 3. Two angles of triangle RST have the same measure. Without measuring, determine the measures of $\angle S$ and $\angle T$.



a. $m\angle R =$ 30°.

b. $m\angle S =$ ______.

c. $m\angle T =$ ______.

Property 2. Given a triangle like the one in Fig. 8.1, $m\angle r + m\angle s = m\angle u$.

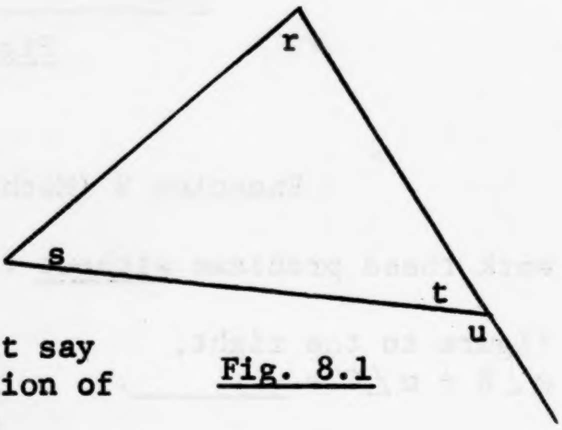


Fig. 8.1

Notice that this does not say that $m\angle s + m\angle t = m\angle u$, nor does it say that $m\angle r + m\angle t = m\angle u$. The position of the two angles whose sum is $m\angle u$ is important.

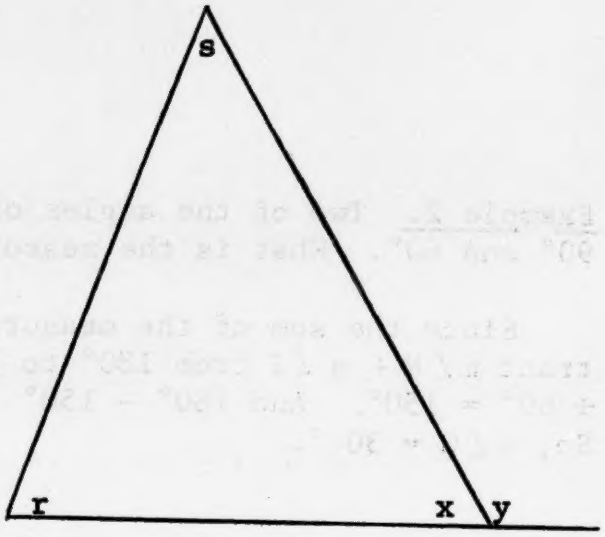


Fig. 8.2

Example 1. In Fig. 8.2, suppose m∠r = 70° and m∠s = 50°. What is m∠y? ________

By Property 2, m∠y = m∠r + m∠s. Now, m∠r + m∠s = 70° + 50° = 120°. So m∠y = 120°.

Problem 1. What is the measure of ∠d in Fig. 8.3? ________

Problem 2. Use Property 1 from the first part of this lesson to find m∠e in Fig. 8.4.________

Now use Property 2 to find m∠h. ______

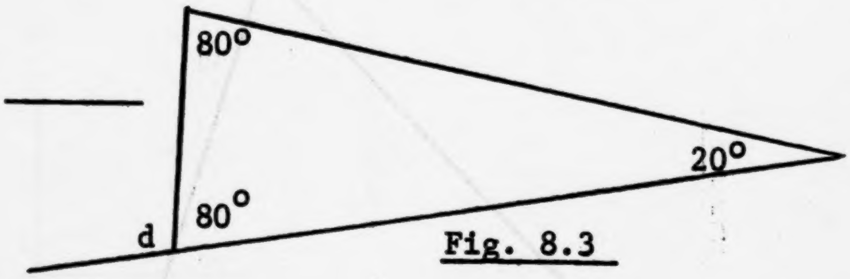


Fig. 8.3

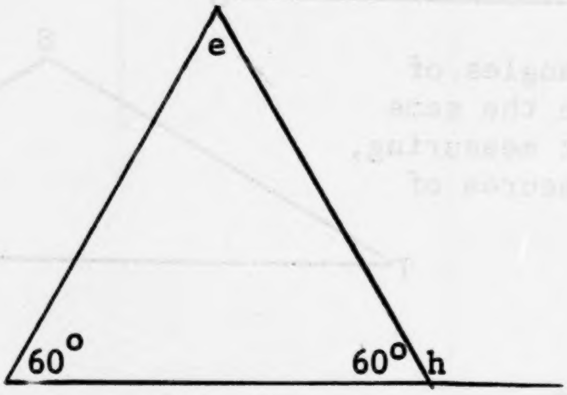


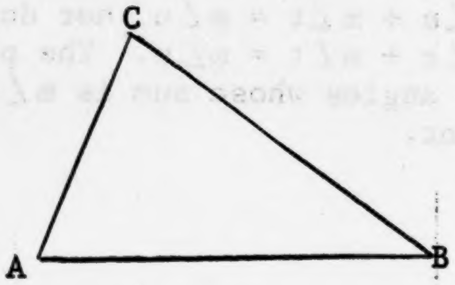
Fig. 8.4

Exercise 8 (Method D, E, and S)

You are to work these problems without the use of a protractor.

1. In the figure to the right, m∠A + m∠B + m∠C = ________.

2. In the figure to the right, suppose m∠A = 70° and m∠B = 30°. What is the measure of ∠C? ________



3. In the figure above, suppose $m\angle A = 40^\circ$ and $m\angle B = 90^\circ$. What is $m\angle C$? ________

4. Is it possible for $m\angle A = 100^\circ$ and $m\angle B = 100^\circ$? ________

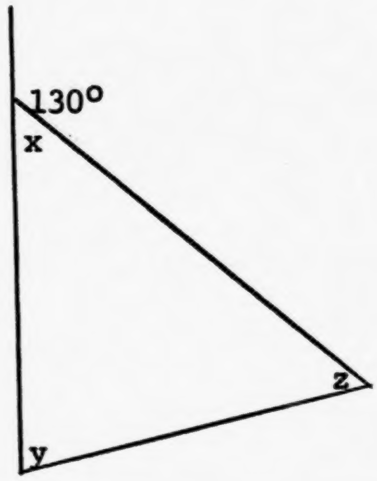
   Explain your answer.________________________

5. In the figure to the right,

   a. $m\angle x + m\angle y + m\angle z =$ ________.

   b. $m\angle y + m\angle z =$ ________.

   c. $m\angle x =$ ________.

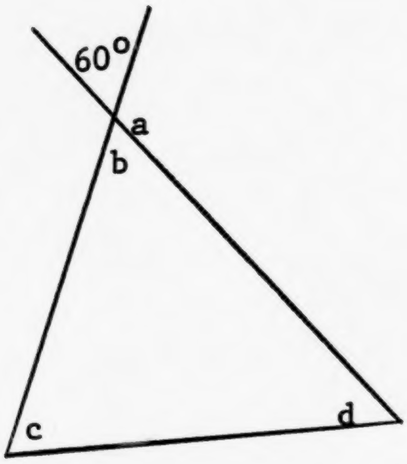


6. In the figure below,

   a. $m\angle a =$ ________.

   b. $m\angle c + m\angle d =$ ________.

   c. $m\angle b =$ ________.



APPENDIX C

Tables

TABLE 3

WEIGHTS FOR THE INITIAL FORMS OF THE OBSERVER RATING SCALE (ORS) AND STUDENT RATING SCALE (SRS)

Item	Method D	Method E	Method I
1	+1	-1	0
2	-1	-1	+1
3	-1	+1	-1
4	+1	0	-1
5	-1	+1	0
6	+1	-1	-1
7	0	-1	+1
8	-1	0	+1
9	-1	+1	-1
10	-1	+1	0

TABLE 4

NUMBER OF OBSERVATIONS AND MEAN FOR EACH TEACHER-METHOD GROUP ON THE INITIAL FORM OF THE STUDENT RATING SCALE ON UNIT 1

	METHOD D		METHOD E		METHOD I	
	No. of Observations	Mean	No. of Observations	Mean	No. of Observations	Mean
Teacher 1			10	-0.2	12	+1.1
Teacher 3	4	-1.3	6	+1.3		
Teacher 4	8	-0.1			7	+0.6
Teacher 5	16	+1.3			12	-0.2
Teacher 6	18	+0.6	11	+0.2		
Teacher 7			14	+2.2	8	+2.8

NOTE: (1) Some cells are empty since each teacher taught by two methods.

(2) Data were unavailable on Teacher 2.

TABLE 5

SCORES ON THE INITIAL FORM OF THE OBSERVER RATING SCALE (UNIT 1)

Teacher	Method D	Method E	Method I
1		0	+1
2	+2		+1
3	+1	+1	
4	+1		0
5	+3,+3		-1,0
6	+3,+3	+1,+2	
7		+1,+1	-3.0

NOTE: (1) In each row one cell is empty since each teacher taught by two methods.

(2) Non-empty cells of Teachers 5, 6, and 7 have two scores.

TABLE 6

PROPORTION OF CONFIRMING RESPONSES (C_p) ON THE OBSERVER RATING SCALE FOR UNIT 2 DATA FOR EACH OF THE TWO TEACHING METHODS OF EACH TEACHER

Method D			Method E			Method S		
Teacher	Class Observed	C_p [a]	Teacher	Class Observed	C_p [a]	Teacher	Class Observed	C_p [a]
2	1	3/7 or 0.43	1	9	1/7 or 0.14	1	13	3/7 or 0.43
3	2	2/7 or 0.29	3	10	1/7 or 0.14	2	15	4/7 or 0.57
5	5	4/7 or 0.57	6	11	4/7 or 0.57	5	17	1/7 or 0.14
6	6	4/7 or 0.57	7	12	3/7 or 0.43	7	18	5/7 or 0.71

NOTE: Teacher 4 was not observed.

[a] $C_p = \frac{\text{sum of responses indicating adherence}}{\text{total number of responses}}$. For example, three of the seven responses for Class 1 (Teacher 2) indicated adherence to Method D; thus, $C_p = 3/7 = 0.43$

TABLE 7

PROPORTION OF CONFIRMING RESPONSES (C) ON THE UNIT 2 DATA OF THE OBSERVER RATING SCALE FOR EACH TREATMENT GROUP

Treatments	C_p[a]
D (Classes 1, 2, 5, and 6)	14/28 or 0.50
E (Classes 9, 10, 11, 12)	9/28 or 0.32
S (Classes 13, 15, 17, 18)	13/28 or 0.46

[a] $C_p = \dfrac{\text{sum of responses withing treatment group which indicated adherence}}{\text{total number of responses within treatment group}}$

For example, 14 of the 28 responses for the group taught by Method D indicated adherence to Method D; thus,

$$C_p = 14/28 = 0.50 .$$

TABLE 8

PROPORTION OF CONFIRMING RESPONSES (C_p) AND AGREEMENT PROPORTION (A_p) ON THE STUDENT RATING SCALE IN UNIT 2 FOR EACH CLASS

Classes in Treatments	C_p for Part I[a]	C_p for Part II[a]	A_p for Part II[b]
Method D (Classes 1-6)			
1 (2 raters, 4 occasions)	24/72 or 0.33	2/8 or 0.25	2/4 or 0.50
2 (2 raters, 1 occasion each)	5/18 or 0.28	2/2 or 1.00	1/1 or 1.00
3 (2 raters, 2 occasions each)	15/36 or 0.42	3/4 or 0.75	1/2 or 0.50
4 (2 raters, 3 occasions each)	27/54 or 0.50	4/6 or 0.67	1/3 or 0.33
5 (2 raters, 4 occasions each)	53/72 or 0.74	8/8 or 1.00	4/4 or 1.00
6 (2 raters, 4 occasions each)	48/72 or 0.67	8/8 or 1.00	4/4 or 1.00
Method E (Classes 7-12)			
7 (2 raters, 3 occasions each)	23/54 or 0.43	1/6 or 0.17	2/3 or 0.67
8 (2 raters, 3 occasions each)	30/54 or 0.56	1/6 or 0.17	2/3 or 0.67
9 (2 raters, 2 occasions each)	21/36 or 0.58	0/4 or 0	2/2 or 1.00
10 (2 raters, 3 occasions each)	23/54 or 0.43	2/6 or 0.33	1/3 or 0.33
11 (2 raters, 2 occasions each)	11/36 or 0.31	1/4 or 0.25	1/2 or 0.50
12 (2 raters, 4 occasions each)	43/72 or 0.60	5/8 or 0.63	1/4 or 0.25

TABLE 8 (continued)

Classes in Treatments	C_p for Part I[a]	C_p for Part II[a]	A_p for Part II[b]
Method S (Classes 13-18)			
13 (2 raters, 4 occasions each)	48/72 or 0.67	8/8 or 1.00	4/4 or 1.00
14 (2 raters, 4 occasions each)	54/72 or 0.75	1/8 or 0.13	3/4 or 0.75
15 (2 raters, 4 occasions each)	46/72 or 0.64	7/8 or 0.88	3/4 or 0.75
16 (2 raters, 3 occasions each)	27/54 or 0.50	4/6 or 0.67	1/3 or 0.33
17 (2 raters, 4 occasions each)	37/72 or 0.51	5/8 or 0.63	1/4 or 0.25
18 (2 raters, 4 occasions each)	43/72 or 0.60	8/8 or 1.00	4/4 or 1.00

---

[a] $C_p = \frac{\text{sum of responses indicating adherence.}}{\text{total number of responses}}$ For example, 24 of the 72 responses on Part I for Class 1 indicated adherence to Method D; thus for Part I, $C_p = 24/72$ or 0.33. On Part II for Class 1, two of the eight responses indicated adherence to Method D; thus, for Part II, $C_p = 2/8$ or 0.25.

[b] $A_p = \frac{\text{number of rating occasions on which both raters indicated adherence or both indicated non-adherence.}}{\text{total number of rating occasions}}$

For example, on two of the four rating occasions in Class 1, both raters indicated non-adherence to Method D. On the other two occasions they disagreed, one rater indicated adherence and the other rater indicated non-adherence. Thus, $A_p = 2/4$ or 0.50.

TABLE 9

PROPORTION OF CONFIRMING RESPONSES (C_p) ON THE STUDENT RATING

SCALE IN UNIT 2 FOR EACH TREATMENT GROUP

Treatments	C_p for Part I[a]	C_p for Part II[a]
D (Classes 1, 2, 3, 4, 5, and 6)	172/324 or 0.53	27/36 or 0.75
E (Classes 7, 8, 9, 10, 11 and 12)	151/306 or 0.49	10/34 or 0.29
S (Classes 13, 14, 15, 16, 17 and 18)	255/414 or 0.62	33/46 or 0.72

[a] $C_p = \dfrac{\text{sum of responses within treatment group which indicated adherence.}}{\text{total number of responses within treatment group}}$

For example, 172 or the 324 responses on Part I for the group taught by Method D indicated adherence to Method D; thus for Part I, C_p = 172/324 or 0.53. On Part II for the group taught by Method D, 27 or the 36 responses indicated adherence to Method D; thus, for Part II, C_p = 27/36 or o.75.

TABLE 10

WEIGHTS FOR THE UNIT 3 STUDENT RATING SCALE

Item	Method D					Method E					Method I				
	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E
1	2	1	0	-1	-2	-2	1	2	1	-2	-2	1	2	1	-2
2	2	1	0	-1	-2	-2	1	2	1	-2	-2	1	2	1	-2
3	-2	-1	0	1	2	2	1	0	-1	-2	2	1	0	-1	-2
4	-2	-1	0	1	2	-2	-1	0	1	2	2	1	0	-1	-2
5	-2	-1	0	1	2	-2	-1	0	1	2	2	1	0	-1	-2
6	1	2	0	-1	-2	1	2	0	-1	-2	1	2	0	-1	-2
7	-2	-1	0	1	2	2	1	0	-1	-2	-2	1	0	1	2
8	-2	-1	0	1	2	2	1	0	-1	-2	2	1	0	-1	-2
9	-2	-1	0	1	2	2	1	0	-1	-2	-2	-1	0	1	2
10	-2	-1	0	1	2	2	1	0	-1	-2	-1	1	2	-1	-1
11	-2	-1	0	1	2	-2	-1	0	1	2	2	1	0	-1	-2
12	-2	-1	0	1	2	-1	0	2	0	-1	-1	0	2	0	-1
13	-2	-1	0	1	2	-2	1	2	1	-2	2	1	0	-1	-2
14	-2	-1	0	1	2	2	1	0	-1	-2	-2	-1	0	1	2
15	2	1	0	-1	-2	-2	-1	0	1	2	-2	-1	0	1	2

TABLE 11

WEIGHTS FOR THE UNIT 3 OBSERVER RATING SCALE

Item	Method D					Method E					Method I				
	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E
1	2	1	0	-1	-2	-2	1	2	1	-2	-2	1	2	1	-2
2	2	1	0	-1	-2	-2	1	2	1	-2	-2	1	2	1	-2
3	-2	-1	0	1	2	2	1	0	-1	-2	2	1	0	-1	-2
4	-2	-1	0	1	2	-2	-1	0	1	2	2	1	0	-1	-2
5	-2	-1	0	1	2	-2	-1	0	1	2	2	1	0	-1	-2
6	1	2	0	-1	-2	1	2	0	-1	-2	1	2	0	-1	-2
7	-2	-1	0	1	2	2	1	0	-1	-2	-2	-1	0	1	2
8	-2	-1	0	1	2	2	1	0	-1	-2	2	1	0	-1	-2
9	-2	-1	0	1	2	2	1	0	-1	-2	-2	-1	0	1	2
10	-2	-1	0	1	2	2	1	0	-1	-2	-1	1	2	-1	-1
11	-2	-1	0	1	2	-2	-1	0	1	2	2	1	0	-1	-2
12	-2	-1	0	1	2	-1	0	2	0	-1	-1	0	2	0	-1
13	-2	-1	0	1	2	-2	-1	0	1	2	2	1	0	-1	-2
14	-2	-1	0	1	2	2	1	0	-1	-2	-2	-1	0	1	2
15	2	1	0	-1	-2	-2	-1	0	1	2	-2	-1	0	1	2
16	-2	-1	0	1	2	2	1	0	-1	-2	-2	-1	0	1	2

TABLE 12

NUMBER OF OBSERVATIONS, MEANS AND STANDARD DEVIATIONS ON SRS FOR EACH TEACHER-METHOD FOR UNIT 3

	Method D		Method E		Method I	
	No. of Obser-vations	Mean Score	No. of Obser-vations	Mean Score	No. of Obser-vations	Mean Score
Teacher 1			16	0.3	14	0.5
Teacher 2	8	0.8				*
Teacher 3	4	0.2	8	0.4		
Teacher 4	4	1.0			6	0.3
Teacher 5	2	1.4			6	0.4
Teacher 6		*		*		
Teacher 7			14	0.6	11	0.4

*Data not available

NOTE: (1) Empty cell indicates teacher was not assigned the method.

(2) All scores tend more toward the largest possible score (+2) than to the least possible score (approximately -2).

TABLE 13

SCORES ON ORS FOR EACH TEACHER-METHOD FOR UNIT 3

Observer	Teacher	Method D	Method E	Method I
A	1		0.5	0.8
A	2	1.7		0.8
A	3	1.4	0.3	
A	4	1.2		0.2
B	5	1.8		0.2
B	6	1.7	1.0	
B	7		0.8	0.3

NOTE: (1) All teacher-method scores confirm adherence of teacher to assigned method.

(2) Empty cell indicates teacher not assigned this method of instruction.

TABLE 14

SUMMARY OF ANALYSES OF UNIT 1 ACHIEVEMENT DATA FOR WHICH $F < F_{.05}$

Source of Variation	df_1	df_2	F	Adjusted Means
Student sex	1	383	0.97	F=11.22 M=11.58
Grade level	1	385	0.48	8th=11.48 9th=11.19
Period of day	5	369	2.01	$P_1=11.68$ $P_2=11.59$ $P_3=11.05$ $P_4=11.93$ $P_5=10.97$ $P_6=9.59$
Treatment within Male student	2	181	1.87	D=10.60 E=11.70 S=11.48
Treatment within School 2	2	138	2.45	D=10.87 E=12.49 S=11.68
Treatment within 8th grade boys	2	123	2.21	D=11.03 E=12.56 S=11.97
Treatment within 9th grade boys*	2	46	0.38	D=9.45 E=9.97 S=10.32
Treatment within male students of female teachers	2	93	1.71	D=11.41 E=12.82 S=11.33
Treatment within female students of male teachers*	2	68	2.55	D= 9.34 E=11.66 S=11.55

*Heterogeneity of regression significant at .05 level in this analysis.

TABLE 15

SUMMARY OF ANALYSES OF UNIT 1 RETENTION DATA FOR WHICH $F < F_{.05}$

Source of Variation	df_1	df_2	F	Adjusted Means
Student sex	1	383	0.16	F=11.53 M=11.68
Grade level	1	385	1.46	8th=11.75 9th=11.22
Treatment within male students	2	181	1.11	D=10.81 E=11.53 S=11.85
Treatment within 8th grade boys	2	123	1.03	D=11.20 E=12.37 S=12.19
Treatment within 9th grade boys	2	46	0.88	D= 9.13 E=10.38 S=10.88
Treatment within male students of female teachers	2	93	1.56	D=11.33 E=13.16 S=12.53
Treatment within male students of male teachers	2	76	1.47	D= 9.41 E=10.38 S=11.16

Table 16

ANALYSIS OF COVARIANCE OF UNIT 1 ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY TREATMENT

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	2274.144	758.048	64.493 **
Treatment Means	2	233.263	116.631	9.923 **
Heterogeneity of Regression	6	228.452	38.075	3.329 **
Error Sum of Squares	381	4478.223	11.754	
Total Sum of Squares	392	7214.082		

**Significant at .01 level

TABLE 17

ANALYSIS OF VARIANCE OF THE UNIT 1 ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY PRIOR GENERAL MATHEMATICAL ACHIEVEMENT LEVEL AND TEACHER

Source of Variation	df	Sum of Squares	Mean Square	F
Prior Achievement	2	1581.32	790.66	58.92**
Teacher	6	499.08	83.18	6.20**
Prior Achievement x Teacher	12	109.77	9.15	0.68
Model	20	2235.37	111.77	8.33**
Error	371	4978.55	13.42	
Total	391	7213.92		

**Significant at .01 level

TABLE 18

RESULTS OF THE RANGE TESTS WHEN ACHIEVEMENT DATA OF UNIT 1 WERE GROUPED BY METHOD

	$M_D = 10.28$	$M_E = 11.91$	$M_I = 11.92$
M_D	-	**	**
M_E	-	-	ns

**Significant at .01 level.

ns Not significant at .05 level.

TABLE 19

ANALYSIS OF VARIANCE OF UNIT 1 ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY METHOD AND TEACHER

Source of Variation	df	Sum of Squares	Mean Square	F
Treatment (Method)	2	73.73	36.87	2.27
Teacher	6	490.00	81.67	5.04**
Method x Teacher	6	483.52	80.59	4.97**
Model	14	1101.53	78.68	4.83**
Error	377	6112.39	16.21	
Total	391	7213,92		

** Significant at .01 level

TABLE 20

ANALYSIS OF VARIANCE OF THE UNIT 1 ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY METHOD AND MENTAL ABILITY

Source of Variation	df	Sum of Squares	Mean Square	F
Method	2	244.77	122.38	8.72 **
Mental Ability	2	1565.55	782.77	55.75 **
Method x Mental Ability	4	142.69	35.67	2.54 *
Model	8	1836.25	229.53	16.35 **
Error	383	5377.67	14.04	
Total	391	7213.92		

** Significant at .01 level

* Significant at .05 level

TABLE 21

ANALYSIS OF VARIANCE OF THE UNIT 1 ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY METHOD AND PRIOR ACHIEVEMENT IN UNIT 1 (UNIT 1 PRETEST)

Source of Variation	df	Sum of Squares	Mean Square	F
Method	2	168.79	84.39	5.09 **
Pretest FGP-A	2	561.91	280.95	16.96 **
Method x Pretest FGP-A	4	177.86	44.65	2.68 *
Model	8	867.78	108.47	6.55 **
Error	383	6346.14	16.57	
Total	391	7213.92		

** Significant at .01 level
* Significant at .05 level

TABLE 22

ANALYSIS OF COVARIANCE OF THE UNIT 1 RETENTION DATA WHEN SUBJECTS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Means Square	F
Regression	3	2536.748	845.583	62.612 **
Method	2	246.583	123.291	9.129 **
Heterogeneity of Regression	6	51.301	8.550	0.633
Error Sum of Squares	381	5145.445	13.505	
Total Sum of Squares	392	7980.078		

** Significant at .01 level

TABLE 23

RESULTS OF RANGE TESTS ON UNIT 1 RETENTION DATA
WHEN THE SUBJECTS WERE GROUPED BY METHOD

	M_D = 10.46	M_E= 11.96	M_I= 12.31
M_D	-	**	**
M_E	-	-	ns

** Significant at .01 level

ns Not significant at .05 level

TABLE 24

ANALYSIS OF COVARIANCE OF UNIT 1 ACHIEVEMENT DATA WHEN THE SUBJECTS WERE GROUPED BY TEACHER

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	2274.144	758.048	64.352 **
Teacher Group Means	6	293.489	48.915	4.152 **
Heterogeneity of Regression	18	346.854	19.270	1.636
Error Sum of Squares	365	4299.594	11.780	
Total Sum of Squares	392	7214.082		

** Significant at .01 level

TABLE 25

RESULTS OF RANGE TESTS WHEN UNIT 1 ACHIEVEMENT DATA WERE GROUPED BY TEACHER

	T_1	T_2	T_3	T_4	T_5	T_6	T_7
T_1= 11.96		*	**	ns	ns	ns	ns
T_2= 10.19			ns	ns	ns	*	*
T_3= 9.50				*	**	**	**
T_4= 11.16					ns	ns	ns
T_5= 11.70						ns	ns
T_6= 12.10							ns
T_7= 11.99							

** Significant at .01 level

* Significant at .05 level

ns Not significant at .05 level

3

TABLE 26

ANALYSIS OF COVARIANCE OF UNIT 1 RETENTION DATA WHEN THE SUBJECTS
WERE GROUPED BY TEACHER

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	2536.748	845.583	69.733 **
Teacher Group Means	6	556.601	94.433	7.788 **
Heterogeneity of Regression	18	450.737	25.041	2.065 **
Error Sum of Squares	365	4425.992	12.126	
Total Sum of Squares	392	7980.078		

** Significant at .01 level

TABLE 27

ANALYSIS OF VARIANCE OF UNIT 1 RETENTION DATA WHEN SUBJECTS WERE GROUPED BY TEACHER AND PRIOR GENERAL MATHEMATICAL ACHIEVEMENT LEVEL

Source of Variation	df	Sum of Squares	Mean Square	F
Achievement	2	1938.56	969.28	71.02 **
Teacher	6	714.13	119.02	8.72 **
Achievement x Teacher	12	275.41	22.95	1.68
Model	20	2910.57	145.53	10.66 **
Error	371	5063.74	13.65	
Total	391	7974.32		

* Significant at .01 level

TABLE 28

RESULTS OF THE A POSTERIORI TESTS OF THE UNIT 1 RETENTION DATA WHEN STRATIFIED BY TEACHER

	T_1	T_2	T_3	T_4	T_5	T_6	T_7
$T_1 = 11.34$		*	**	ns	*	ns	**
$T_2 = 9.65$			ns	**	**	**	**
$T_3 = 9.52$				**	**	**	**
$T_4 = 11.55$					*	ns	**
$T_5 = 12.95$						*	ns
$T_6 = 11.73$							**
$T_7 = 14.07$							

** Significant at .01 level

* Significant at .05 level

ns Not significant at .05 level

TABLE 29

ANALYSIS OF VARIANCE OF UNIT 1 RETENTION DATA WHEN SUBJECTS WERE GROUPED BY TEACHER AND MENTAL ABILITY LEVEL

Source of Variation	df	Sum of Squares	Mean Square	F
Teacher	6	438.89	73.15	4.76 **
Mental Ability	2	1227.64	613.82	39.96 **
Teacher x Mental Ability	12	351.32	29.28	1.91 *
Model	20	2275.56	113.78	7.41 **
Error	371	5698.76	15.36	
Total	391	7974.32	20.39	

** Significant at .01 level

* Significant at .05 level

TABLE 30

ANALYSIS OF COVARIANCE OF UNIT 1 RETENTION DATA WHEN THE SUBJECTS WERE GROUPED BY PERIOD OF DAY

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	2536.748	845.583	62.011 **
Period Group Means	5	232.946	46.589	3.417 **
Heterogeneity of Regression	15	178.664	11.911	0.873
Error Sum of Squares	369	5031.719	13.636	
Total Sum of Squares	392	7980.078		

** Significant at .01 level

TABLE 31

RESULTS OF RANGE TESTS OF UNIT 1 RETENTION DATA WHEN SUBJECTS WERE GROUPED BY PERIOD OF DAY

	P_1	P_2	P_3	P_4	P_5	P_6
$P_1 = 12.39$		*	*	ns	*	**
$P_2 = 11.18$			ns	*	ns	ns
$P_3 = 11.24$				ns	ns	ns
$P_4 = 12.34$					*	**
$P_5 = 10.46$						ns
$P_6 = 10.00$						

** Significant at .01 level

* Significant at .05 level

ns Not significant at .05 level

TABLE 32

ANALYSIS OF COVARIANCE OF UNIT 1 ACHIEVEMENT DATA WHEN THE SUBJECTS WERE GROUPED BY SCHOOL

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	2274.144	758.048	60.958 **
School Group Means	1	53.673	53.673	4.316 *
Heterogeneity of Regression	3	98.579	32.860	2.642
Error Sum of Squares	385	4787.684	12.436	
Total Sum of Squares	392	7214.082		

** Significant at .01 level

* Significant at .05 level

TABLE 33

ANALYSIS OF COVARIANCE OF UNIT 1 RETENTION DATA WHEN SUBJECTS WERE GROUPED BY SCHOOL

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	2536.748	845.583	64.960 **
School Group Means	1	335.803	335.803	25.797 **
Heterogeneity of Regression	3	95.955	31.985	2.457
Error Sum of Squares	385	5011.570	13.017	
Total Sum of Squares	392	7980/078		

** Significant at .01 level

TABLE 34

ANALYSIS OF COVARIANCE OF THE UNIT 1 ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY METHOD WITHIN EIGHTH GRADE

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1359.087	453.029	36.213 **
Treatment Means	2	174.932	87.466	6.992 **
Heterogeneity of Regression	6	158.305	26.384	2.109
Error Sum of Squares	272	3402.786	12.510	
Total Sum of Squares	283	5095.109		

** Significant at .01 level

TABLE 35

RESULTS OF RANGE TESTS OF EIGHTH GRADE STUDENT ACHIEVEMENT DATA ON UNIT 1 WHEN GROUPED BY METHOD

	M_D	M_E	M_I
$M_D = 10.59$		**	**
$M_E = 12.21$			ns
$M_I = 12.30$			

** Significant at .01 level

ns Not significant at .05 level

TABLE 36

ANALYSIS OF COVARIANCE OF THE UNIT 1 RETENTION DATA WHEN SUBJECTS WERE GROUPED BY METHOD WITHIN EIGHTH GRADE

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1754.044	584.681	41.969 **
Treatment Means	2	185.310	92.655	6.635 **
Heterogeneity of Regression	6	58.284	9.714	0.696
Error Sum of Squares	272	3798.346	13.965	
Total Sum of Squares	283	5795.984		

** Significant at .01 level

TABLE 37

ANALYSIS OF COVARIANCE OF THE UNIT 1 ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY METHOD WITHIN NINTH GRADE

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	899.594	299.865	33.494 **
Treatment Means	2	77.827	38.913	4.347 *
Heterogeneity of Regression	6	187.213	31.202	3.485 **
Error Sum of Squares	97	868.413	8.953	
Total Sum of Squares	108	2033.047		

** Significant at .01 level

* Significant at .05 level

TABLE 38

ANALYSIS OF COVARIANCE OF THE UNIT 1 RETENTION DATA WHEN SUBJECTS WERE GROUPED BY METHOD WITHIN NINTH GRADE

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	728.008	242.669	20.073 **
Treatment Means	2	79.792	39.896	3.300 *
Heterogeneity of Regression	6	87.271	14.545	1.203
Error Sum of Squares	97	1172.675	12.089	
Total Sum of Squares	108	2067.746		

** Significant at .01 level

* Significant at .05 level

TABLE 39

ANALYSIS OF COVARIANCE OF THE UNIT 1 ACHIEVEMENT DATA WHEN SUBJECTS OF FEMALE TEACHERS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	709.601	236.534	19.394 **
Treatment Means	2	120.727	60.364	4.949 **
Heterogeneity of Regression	6	153.239	25.540	2.094
Error Sum of Squares	211	2573.351	12.196	
Total Sum of Squares	222	3556.918		

** Significant at .01 level

TABLE 40

ANALYSIS OF COVARIANCE OF THE UNIT 1 RETENTION DATA WHEN SUBJECTS OF FEMALE TEACHERS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1052.067	350.689	25.804 **
Treatment Means	2	246.157	123.079	9.056 **
Heterogeneity of Regression	6	109.483	18.247	1.343
Error Sum of Squares	211	2867.594	13.590	
Total Sum of Squares	222	4275.301		

** Significant at .01 level

TABLE 41

ANALYSIS OF COVARIANCE OF THE UNIT 1 ACHIEVEMENT DATA WHEN SUBJECTS OF MALE TEACHERS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1595.269	531.756	48.942 **
Treatment Means	2	121.534	60.767	5.593 **
Heterogeneity of Regression	6	169.661	28.277	2.603 *
Error Sum of Squares	157	1705.821	10.865	
Total Sum of Squares	168	3592.285		

** Significant at .01 level
* Significant at .05 level

TABLE 42

ANALYSIS OF COVARIANCE OF THE UNIT 1 RETENTION DATA WHEN SUBJECTS OF MALE TEACHERS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1412.511	470.837	40.411 **
Treatment Means	2	88.244	44.122	3.790 *
Heterogeneity of Regression	6	81.575	13.596	1.168
Error Sum of Squares	157	1827.896	11.643	
Total Sum of Squares	168	3410.227		

** Significant at .01 level

* Significicat at .05 level

TABLE 43

ANALYSIS OF COVARIANCE OF THE FEMALE STUDENT ACHIEVEMENT DATA ON UNIT 1 WHEN GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1035.464	345.155	26.824 **
Treatment Means	2	240.486	120.243	9.345 **
Heterogeneity Of Regression	6	160.181	26.297	2.075
Error Sum of Squares	186	2393.348	12.867	
Total Sum of Squares	197	3829.480		

** Significant at .01 level

TABLE 44

ANALYSIS OF COVARIANCE OF THE FEMALE STUDENT RETENTION DATA ON UNIT 1 WHEN GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1376.669	458.889	39.057 **
Treatment Means	2	258.462	128.231	10.999 **
Heterogeneity of Regression	6	29.191	4.865	0.414
Error Sum of Squares	186	2185.343	11.749	
Total Sum of Squares	197	3849.664		

** Significant at .01 level

TABLE 45

ANALYSIS OF COVARIANCE OF THE SCHOOL 1 ACHIEVEMENT DATA ON UNIT 1 WHEN THE SUBJECTS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1924.686	641.562	61.772 **
Treatment Means	2	200.212	100.106	9.639 **
Heterogeneity of Regression	6	206.124	34.354	3.308 **
Error Sum of Squares	231	2399.162	10.386	
Total Sum of Squares	242	4730.184		

** Significant at .01 level

TABLE 46

ANALYSIS OF COVARIANCE OF THE SCHOOL 1 RETENTION DATA ON UNIT 1
WHEN THE SUBJECTS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1749.989	583.330	47.801 **
Treatment Means	2	104.550	52.275	4.284 *
Heterogeneity of Regression	6	92.850	15.475	1.268
Error Sum of Squares	231	2818.966	12.203	
Total Sum of Squares	242	4766.355		

** Significant at .01 level

* Significant at .05 level

TABLE 47

ANALYSIS OF COVARIANCE OF THE SCHOOL 2 RETENTION DATA ON UNIT 1 WHEN THE SUBJECTS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1045.869	348.623	29.688 **
Treatment Means	2	216.278	108.139	9.209 **
Heterogeneity Of Regression	6	158.383	26.397	2.248 *
Error Sum of Squares	138	1620.545	11.743	
Total Sum of Squares	149	3041.074		

** Significant at .01 level
 * Significant at .05 level

TABLE 48

ANALYSIS OF COVARIANCE OF THE EIGHTH GRADE FEMALE STUDENT ACHIEVEMENT DATA ON UNIT 1 WHEN GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	664.700	221.567	15.605 **
Treatment Means	2	158.080	79.040	5.567 **
Heterogeneity of Regression	6	90.854	15.142	1.067
Error Sum of Squares	135	1916.737	14.198	
Total Sum of Squares	146	2830.371		

** Significant at .01 level

TABLE 49

ANALYSIS OF COVARIANCE OF THE EIGHTH GRADE FEMALE STUDENT RETENTION DATA ON UNIT 1 WHEN GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	764.887	254.962	20.774 **
Treatment Means	2	186.127	93.063	7.583 **
Heterogeneity 6f Regression	6	31.990	5.332	0.434
Error Sum of Squares	135	1656.891	12.273	
Total Sum of Squares	146	2639.895		

** Significant at .01 level

TABLE 50

ANALYSIS OF COVARIANCE OF THE NINTH GRADE FEMALE STUDENT ACHIEVEMENT DATA ON UNIT 1 WHEN GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	428.397	142.799	19.068 **
Treatment Means	2	124.159	62.080	8.290 **
Heterogeneity of Regression	6	153.538	25.590	3.417 **
Error Sum of Squares	39	292.066	7.489	
Total Sum of Squares	50	998.160		

** Significant at .01 level

TABLE 51

ANALYSIS OF COVARIANCE OF THE NINTH GRADE FEMALE STUDENT RETENTION DATA ON UNIT 1 WHEN SUBJECTS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	612.403	204.134	17.594 **
Treatment Means	2	81.356	40.678	3.506 *
Heterogeneity of Regression	6	42.931	7.155	0.617
Error Sum of Squares	39	452.489	11.602	
Total Sum of Squares	50	1189.180		

** Significant at .01 level
 * Significant at .05 level

TABLE 52

ANALYSIS OF COVARIANCE OF THE FEMALE STUDENT OF FEMALE TEACHERS ACHIEVEMENT DATA IN UNIT 1 WHEN GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	255.674	85.225	6.486 **
Treatment Means	2	151.843	75.992	5.778 **
Heterogeneity of Regression	6	80.549	13.425	1.022
Error Sum of Squares	106	1392.794	13.140	
Total Sum of Squares	117	1880.859		

** Significant at .01 level

TABLE 53

ANALYSIS OF COVARIANCE OF THE FEMALE STUDENTS OF FEMALE TEACHERS RETENTION DATA IN UNIT 1 WHEN GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	410.349	136.783	12.704 **
Treatment Means	2	224.101	112.051	10.407 **
Heterogeneity of Regression	6	19.632	3.272	0.304
Error Sum of Squares	106	1141.285	10.767	
Total Sum of Squares	117	1795.367		

** Significant at .01 level

TABLE 54

ANALYSIS OF COVARIANCE OF THE FEMALE STUDENT OF MALE TEACHERS RETENTION DATA IN UNIT 1 WHEN GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	959.650	319.883	27.707 **
Treatment Means	2	87.677	43.839	3.797 *
Heterogeneity of Regression	6	85.550	14.258	1.235
Error Sum of Squares	68	785.076	11.545	
Total Sum of Squares	79	1917.953		

** Significant at .01 level
 * Significant at .05 level

TABLE 55

ANALYSIS OF COVARIANCE OF THE MALE STUDENT OF MALE TEACHERS ACHIEVEMENT DATA IN UNIT 1 WHEN GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	761.550	253.850	27.783 **
Treatment Means	2	61.426	30.713	3.361 *
Heterogeneity of Regression	6	129.609	21.601	2.364 *
Error Sum of Squares	76	694.407	9.137	
Total Sum of Squares	87	1646.992		

** Significant at .01 level

* Significant at .05 level

TABLE 56

SUMMARY OF THE HOMOGENEITY OF VARIANCE TEST ON THE ACHIEVEMENT TEST

Source of Variation	df	Sum of Squares	Mean Square	F	$F_{.95}$
Treatments	2	21.9	11.0	2.29	3.68
Groups Within Treatments	15	72.4	4.8		
Total	17	94.3			

TABLE 57

SUMMARY OF THE HOMOGENEITY OF VARIANCE TEST ON THE RETENTION TEST

Source of Variation	df	Sum of Squares	Mean Square	F	$F_{.95}$
Treatments	2	30.9	15.5	3.30	3.68
Groups Within Treatments	15	71.0	4.7		
Total	17	101.9			

TABLE 58

SUMMARY OF THE ANALYSIS OF VARIANCE FOR THE GROUPS-WITHIN-TREATMENTS DESIGN ON THE ACHIEVEMENT TEST

Source of Variation	df	Sum of Squares	Mean Square	F	$F_{.95}$
Treatments	2	2.9	1.5	0.07	3.68
Groups-Within-Treatments	15	305.5	20.4		
Total	17	308.4			

TABLE 59

SUMMARY OF THE ANALYSIS OF VARIANCE FOR THE GROUPS-WITHIN-TREATMENTS DESIGN ON THE RETENTION TEST

Source of Variation	df	Sum of Squares	Mean Square	F	$F_{.95}$
Treatments	2	7.4	3.7	0.14	3.68
Groups-Within-Treatments	15	388.3	25.9		
Total	17	395.7			

TABLE 60

SUMMARY OF THE ANALYSIS OF VARIANCE FOR THE THREE-FACTOR FACTORIAL DESIGN AT THE NINTH GRADE LEVEL ON THE ACHIEVEMENT TEST

Source of Variation	df	Sum of Squares	Mean Square	F	$F_{.95}$
T (treatments)	2	305.33	152.67	1.11	9.55
C:T (classes nested in treatments)	3	413.52	137.84	3.93*	2.68
A (achievement level on the Unit 2 pretest)	1	2813.78	2813.78	57.81*	10.13
TA	2	26.57	13.29	0.27	1.55
AC:T	3	146.01	48.67	1.39	2.68
Error	122	4275.23	35.04		

* Significant at the .05 level.

TABLE 61

NINTH GRADE CLASS TOTALS ON THE UNIT 2 ACHIEVEMENT TEST ARRANGED IN INCREASING ORDER OF MAGNITUDE

Class Total	Order	Class
403.5	1	2
413.0	2	13
485.3	3	1
488.2	4	10
505.8	5	15
553.4	6	8

TABLE 62

SUMMARY OF THE ANALYSIS OF VARIANCE FOR THE THREE-FACTOR FACTORIAL DESIGN AT THE NINTH GRADE LEVEL ON THE UNIT 2 RETENTION TEST

Source of Variation	df	Sum of Squares	Mean Square	F	$F_{.95}$
T (treatments)	2	1336.26	668.13	5.61	9.55
C:T(classes nested in treatments)	3	357.08	119.03	2.73*	2.68
A (achievement level on the pretest)	1	2711.02	2711.02	73.99*	10.13
TA	2	35.60	17.80	0.49	9.55
AC:T	3	109.92	36.64	0.84	2.68
Error	122	5313.15	43.55		

* Significant at the .05 level.

TABLE 63

NINTH GRADE CLASS TOTALS ON THE UNIT 2 RETENTION TEST ARRANGED IN INCREASING ORDER OF MAGNITUDE

Class Total	Order	Class
361.1	1	13
370.1	2	2
427.1	3	15
469.7	4	1
545.4	5	8
548.2	6	10

TABLE 64

SUMMARY OF THE ANALYSIS OF VARIANCE FOR THE FOUR-FACTOR FACTORIAL DESIGN AT THE EIGHTH GRADE LEVEL ON THE UNIT 2 ACHIEVEMENT TEST

Source of Variation	df	Sum of Squares	Mean Square	F	$F_{.95}$
T (treatments)	2	214.41	107.20	0.66	5.14
S (schools)	1	37.82	37.82	0.23	5.99
C:TS (classes nested in treatments and schools)	6	969.79	161.63	4.44*	2.13
A (achievement level on the Unit 2 pretest)	1	1956.32	1956.32	35.35*	5.99
TS	2	32.05	16.03	0.10	5.14
TA	2	99.86	49.93	0.90	5.14
SA	1	50.94	50.94	0.92	5.99
AC:TS	6	322.01	55.34	1.56	2.13
TSA	2	2.67	1.33	0.02	5.14
Error	253	9208.67	36.40		

* Significant at the .05 level.

TABLE 65

EIGHTH GRADE CLASS TOTALS ON THE UNIT 2 ACHIEVEMENT TEST ARRANGED IN INCREASING ORDER OF MAGNITUDE

Class Total	Order	Class
258.5	1	9
300.6	2	18
332.2	3	11
344.9	4	3
347.0	5	7
347.6	6	5
353.9	7	4
355.7	8	6
348.6	9	14
392.6	10	16
405.3	11	12
420.7	12	17

TABLE 66

SUMMARY OF THE ANALYSIS OF VARIANCE FOR THE FOUR-FACTOR FACTORIAL DESIGN AT THE EIGHTH GRADE LEVEL ON THE UNIT 2 RETENTION TEST

Source of Variation	df	Sum of Squares	Mean Square	F	$F_{.95}$
T (treatments)	2	282.03	141.01	0.55	5.14
S (schools)	1	45.94	45.94	0.18	5.99
C:TS (classes nested in treatments and schools)	6	1548.05	258.01	7.55*	2.13
A (achievement level on the Unit 2 pretest)	1	2102.07	2102.07	12.97	5.99
TS	2	90.68	45.34	0.18	5.14
TA	2	50.48	25.24	0.16	5.14
SA	1	34.62	34.62	0.21	5.99
AC:TS	6	972.65	162.11	4.75*	2.13
TSA	2	71.23	35.61	0.22	5.14
Error	253	8642.36	34.16		

* Significant at the .05 level

TABLE 67

EIGHTH GRADE CLASS TOTALS ON THE UNIT 2 RETENTION TEST ARRANGED IN INCREASING ORDER OF MAGNITUDE

Class Total	Order	Class
218.3	1	9
277.7	2	18
283.9	3	11
300.1	4	16
343.0	5	6
345.2	6	4
351.6	7	7
353.4	8	5
362.4	9	14
362.5	10	3
372.6	11	12
398.2	12	17

TABLE 68

SUMMARY OF ANALYSES OF UNIT 3 ACHIEVEMENT DATA FOR WHICH $F < F_{.05}$

Source of Variation	df_1	df_2	F	Adjusted Means		
Student Sex	1	396	0.01	F; 18.34		M; 18.30
Grade Level	1	398	0.67	8th; 18.25		9th; 18.59
Treatment within Ninth Grade	2	111	2.35	D; 17.24	E; 18.53	S; 19.04
Treatment within Male Student	2	188	2.83	D; 17.17	E; 18.43	S; 18.45
Treatment within 8th Grade Boys	2	125	1.54	D; 17.59	E; 18.63	S; 18.64
Treatment within 9th Grade Boys	2	051	0.98	D; 17.33	E; 17.88	S; 18.07
Treatment within Male Students of Female Teachers	2	92	2.04	D; 17.60	E; 19.43	S; 18.72
Treatment within Female Students of Male Teachers	2	69	1.96	D; 16.49	E; 18.71	S; 18.58
Treatment within Male Students of Male Teachers	2	83	1.43	D; 16.54	E; 17.74	S; 18.07

TABLE 69

SUMMARY OF ANALYSES OF UNIT 3 RETENTION DATA FOR WHICH $F < F_{.05}$

Source of Variation	df_1	df_2	F	Adjusted Means
Treatment	2	394	0.82	D;16.00 E;16.47 S;16.51
Student Sex	1	396	1.59	F;16.08 M;16.54
Grade Level	1	398	1.20	8th;16.47 9th; 16.01
Treatment within Eighth Grade	2	271	1.39	D;16.55 E;16.25 S;17.13
Treatment within Female Teachers	2	215	1.28	D;16.21 E;16.97 S;17.05
Treatment within Male Teacher	2	165	0.13	D;15.86 E;15.95 S;15.64
Treatment within Male Students	2	188	0.16	D;16.06 E;16.11 S;16.39
Treatment within Female Students	2	192	1.03	D;15.90 E;16.70 S;16.68
Treatment within School 1	2	248	0.57	D;15.83 E;16.40 S;16.01
Treatment within School 2	2	134	1.22	D;16.17 E;16.77 S;17.30
Treatment within 8th Grade Girls	2	132	0.46	D;16.23 E;16.11 S;16.82
Treatment within 8th Grade Boys	2	125	1.37	D;16.89 E;16.25 S;17.48
Treatment within 9th Grade Boys	2	551	2.05	D;13.97 E;16.08 S;13.85
Treatment within Female Students of Female Teachers	2	111	1.31	D;16.13 E;17.59 S;17.22
Treatment within Male Students of Female Teachers	2	92	0.35	D;16.31 E;16.40 S;16.95
Treatment within Female Students of Male Teachers	2	69	1.35	D;15.35 E;16.30 S;14.98
Treatment within Male Students of Male Teachers	2	83	0.17	D;16.14 D;15.55 S;15.74

TABLE 70

ANALYSIS OF COVARIANCE OF THE UNIT 3 ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	6308.984	2102.995	162.380 **
Treatment Means	2	186.562	93.281	7.203 **
Heterogeneity of Regression	6	142.668	23.778	1.836
Error Sum of Squares	394	5102.723	12.951	
Total Sum of Squares	405	11704.937		

** Significant at .01 Level

TABLE 71

RESULTS OF A POSTERIORI TESTS OF UNIT 3 ACHIEVEMENT DATA WHEN STRATIFIED BY METHOD

	M_D	M_E	M_I
M_D= 17.38		**	*
M_E= 18.92			ns
M_I= 18.73			

** Significant at .01 level

* Significant at .05 level

ns Not significant at .05 level

TABLE 72

ANALYSIS OF COVARIANCE OF THE UNIT 3 ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY TEACHER

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Squares	F
Regression	3	6308.984	2102.995	166.822 **
Teacher Group Means	6	355.273	55.879	4.433 **
Heterogeneity of Regression	18	331.527	18.418	1.461
Error Sum of Squares	378	4765.152	12.606	
Total Sum of Squares	405	11740.937		

** Significant at .01 level

TABLE 73

RESULTS OF THE A POSTERIORI TESTS OF UNIT 3 ACHIEVEMENT DATA WHEN STRATIFIED BY TEACHER

	T_1	T_2	T_3	T_4	T_5	T_6	T_7
T_1= 18.60		ns	*	*	ns	ns	ns
T_2= 18.14			ns	ns	*	ns	ns
T_3= 17.15				ns	**	ns	**
T_4= 16.92					*	ns	**
T_5= 19.74						*	ns
T_6= 18.17							ns
T_7= 19.39							

** Significant at .01 level

* Significant at .05 level

ns Not Significant at .05 level

TABLE 74

ANALYSIS OF COVARIANCE OF THE UNIT 3 RETENTION DATA WHEN SUBJECTS
WERE GROUPED BY TEACHER

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	6621.363	2207.121	176.221 **
Teacher Group Means	6	195.258	32.543	2.598 *
Heterogeneity of Regression	18	344.848	19.158	1.530
Error Sum of Squares	378	4734.344	12.525	
Total Sum of Squares	405	11895.812		

** Significant at .01 level

* Significant at .05 level

TABLE 75

ANALYSIS OF COVARIANCE OF THE UNIT 3 ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY PERIOD OF DAY

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	6308.984	2102.995	161.513 **
Period Group Means	5	179.008	35.802	2.750 *
Heterogeneity of Regression	15	279.082	18.605	1.429
Error Sum of Squares	382	4973.863	13.021	
Total Sum of Squares	405	11740.937		

** Significant at .01 level

* Significant at .05 level

TABLE 76

ANALYSIS OF COVARIANCE OF THE UNIT 3 RETENTION DATA WHEN SUBJECTS WERE GROUPED BY PERIOD OF DAY

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	6621.363	2207.121	173.587 **
Period Group Means	5	211.602	42.320	3.328
Heterogeneity of Regression	15	205.789	13.719	1.079
Error Sum of Squares	382	4857.059	12.715	
Total Sum of Squares	405	11895.812		

** Significant at .01 level

TABLE 77

ANALYSIS OF COVARIANCE OF THE UNIT 3 ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY SCHOOL

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	6308.984	2102.995	158.636 **
School Group Means	1	134.687	134.687	10.160 **
Heterogentity of Regression	3	21.090	7.030	0.530
Error Sum of Squares	398	5276.176	13.257	
Total Sum of Squares	405	11740.937		

** Significant at .01 level

TABLE 78

ANALYSIS OF COVARIANCE OF THE UNIT 3 RETENTION DATA WHEN SUBJECTS WERE GROUPED BY SCHOOL

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	6621.363	2207.121	170.700 **
School Group Means	1	105.234	105.234	8.139 **
Heterogeneity of Regression	3	23.141	7.714	0.597
Error Sum of Squares	398	5146.074	12.903	
Total Sum of Squares	405	11895.812		

** Significant at .01 level

TABLE 79

ANALYSIS OF COVARIANCE OF THE UNIT 3 ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY METHOD WITHIN EIGHTH GRADE

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	3893.883	1297.961	105.373 **
Treatment Means	2	114.253	57.126	4.638 *
Heterogeneity of Regression	6	40.060	6.677	0.542
Error Sum of Squares	271	3338.117	12.318	
Total Sum of Squares	282	7386.321		

** Significant at .01 level

* Significant at .05 level

TABLE 80

ANALYSIS OF COVARIANCE OF THE UNIT 3 RETENTION DATA WHEN SUBJECTS WERE GROUPED BY METHOD WITHIN THE NINTH GRADE

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	2229.730	743.243	61.922 **
Treatment Means	2	145.653	72.826	6.067
Heterogeneity of Regression	6	82.695	13.783	1.148
Error Sum of Squares	111	1332.332	12.003	
Total Sum of Squares	122	3790.410		

** Significant at .01 level

TABLE 81

RESULTS OF THE A POSTERIORI TESTS OF NINTH GRADE RETENTION DATA IN UNIT 3 WHEN GROUPED BY METHOD

	M_D	M_E	M_I
$M_D = 14.54$		**	ns
$M_E = 17.01$			*
$M_I = 14.36$			

** Significant at .01 level

* Significant at .05 level

ns Not Significant at .05 level

TABLE 82

ANALYSIS OF COVARIANCE OF THE UNIT 3 ACHIEVEMENT DATA WHEN SUBJECTS OF FEMALE TEACHERS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	2478.550	826.183	62.389 **
Treatment Means	2	197.125	98.563	7.443 **
Heterogeneity of Regression	6	59.027	9.838	0.743
Error Sum of Squares	215	2847.110	13.242	
Total Sum of Squares	226	5581.812		

** Significant at .01 level

TABLE 83

RESULTS OF THE A POSTERIORI TESTS ON UNIT 3 ACHIEVEMENT DATA FOR SUBJECTS OF FEMALE TEACHERS WHEN GROUPED BY METHOD

	M_D	M_E	M_I
$M_D = 17.68$		**	*
$M_E = 20.31$			*
$M_I = 18.88$			

** Significant at .01 level

* Significant at .05 level

TABLE 84

ANALYSIS OF COVARIANCE OF THE UNIT 3 ACHIEVEMENT DATA WHEN SUBJECTS OF MALE TEACHERS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	3844.109	1281.370	105.361 **
Treatment Means	2	80.852	40.426	3.324 *
Heterogeneity of Regression	6	156.895	26.149	2.150 *
Error Sum of Squares	165	2006.687	12.162	
Total Sum of Squares	176	6088.543		

** Significant at .01 level
* Significant at .05 level

TABLE 85

ANALYSIS OF COVARIANCE OF THE FEMALE STUDENT ACHIEVEMENT DATA ON UNIT 3 WHEN GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	3158.174	1052.725	75.890 **
Treatment Means	2	130.293	65.147	4.696 *
Heterogeneity of Regression	6	137.430	22.905	1.651
Error Sum of Squares	192	2663.352	13.872	
Total Sum of Squares	203	6089.250		

** Significant at .01 level
 * Significant at .05 level

TABLE 86

ANALYSIS OF COVARIANCE OF THE SCHOOL 1 ACHIEVEMENT DATA ON UNIT 3 WHEN THE SUBJECTS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	4570.703	1523.568	122.867 **
Treatment Means	2	145.520	72.760	5.868 **
Heterogeneity of Regression	6	259.109	43.185	3.483 **
Error Sum of Squares	248	3075.230	12.400	
Total Sum of Squares	259	8050.562		

** Significant at .01 level

TABLE 87

ANALYSIS OF COVARIANCE OF THE SCHOOL 2 ACHIEVEMENT DATA ON UNIT 3 WHEN SUBJECTS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1866.025	622.008	51.110 **
Treatment Means	2	90.177	45.089	3.705 *
Heterogeneity of Regression	6	75.353	12.559	1.032
Error Sum of Squares	134	1630.781	12.170	
Total Sum of Squares	145	3662.336		

** Significant at .01 level
 * Significant at .05 level

TABLE 88

ANALYSIS OF COVARIANCE OF THE EIGHTH GRADE FEMALE STUDENT ACHIEVEMENT DATE

WHEN GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1834.112	611.371	40.896 **
Treatment Means	2	100.495	50.247	3.361 *
Heterogeneity of Regression	6	20.718	3.453	0.231
Error Sum of Squares	132	1973.316	14.949	
Total Sum of Squares	143	3928.641		

** Significant at .01 level

** Significant at .05 level

TABLE 89

ANALYSIS OF COVARIANCE OF THE NINTH GRADE FEMALE STUDENT ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1360.340	453.447	58.037 **
Treatment Means	2	89.926	44.963	5.755 **
Heterogeneity of Regression	6	305.692	50.949	6.521 **
Error Sum of Squares	48	375.026	7.813	
Total Sum of Squares	59	2130.984		

** Significant at .01 level

TABLE 90

ANALYSIS OF COVARIANCE OF THE NINTH GRADE FEMALE STUDENT RETENTION DATA
ON UNIT 3 WHEN SUBJECTS WERE GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1216.849	405.616	39.954 **
Treatment Means	2	74.700	37.350	3.679 *
Heterogeneity of Regression	6	120.004	20.001	1.970
Error Sum of Squares	48	487.299	10.152	
Total Sum of Squares	59	1898.852		

** Significant at .01 level

** Significant at .05 level

TABLE 91

ANALYSIS OF COVARIANCE OF THE FEMALE STUDENT OF FEMALE TEACHERS ACHIEVEMENT DATA IN UNIT 3 WHEN GROUPED BY METHOD

Source of Variation	df	Adjusted Sum of Squares	Adjusted Mean Square	F
Regression	3	1294.439	431.480	32.198 **
Treatment Means	2	170.078	85.039	6.346 **
Heterogeneity of Regression	6	31.301	5.217	0.389
Error Sum of Squares	111	1487.502	13.401	
Total Sum of Squares	122	2983.320		

** Significant at .01 level

TABLE 92

ANALYSIS OF VARIANCE OF UNIT 3 ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY METHOD AND TEACHER

Source of Variation	df	Sum of Squares	Mean Squares	F
Method	2	163.76	81.88	3.22 *
Teacher	6	672.02	112.00	4.41 **
Method x Teacher	5	941.37	188.27	7.41 **
Model	13	1775.02	136.54	5.37 **
Error	392	9965.91	25.42	
Total	405	11740.09		

** Significant at .01 level

* Significant at .05 level

TABLE 93

ANALYSIS OF VARIANCE OF UNIT 3 ACHIEVEMENT DATA WHEN SUBJECTS WERE GROUPED BY TEACHER AND MENTAL ABILITY LEVEL

Source of Variation	df	Sum of Squares	Mean Square	F
Teacher	6	372.00	62.00	3.06 **
Mental Ability	2	2807.04	1403.52	6.93 **
Teacher x Mental Ability	12	461.88	38.49	1.90 *
Model	20	3938.80	196.94	9.72 **
Error	385	7802.13	20.27	
Total	405	11740.93		

** Significant at .01 level
* Significant at .05 level

TABLE 94

ANALYSIS OF VARIANCE OF UNIT 3 RETENTION DATA WHEN SUBJECTS WERE GROUPED BY METHOD AND TEACHER

Source of Variation	df	Sum of Squares	Mean Square	F
Method	2	41.25	20.62	0.80
Teacher	6	699.06	116.51	4.50 **
Method x Teacher	5	1020.42	204.08	7.88 **
Model	13	1744.86	134.20	5.18 **
Error	392	10150.92	25.90	
Total	405	11895.77		

** Significant at .01 level

TABLE 95

ANALYSIS OF VARIANCE OF UNIT 3 RETENTION DATA WHEN SUBJECTS WERE GROUPED BY METHOD AND MENTAL ABILITY LEVEL

Source of Variation	df	Sum of Squares	Mean Square	F
Method	2	85.31	42.66	2.22
Mental Ability	2	4027.04	2013.52	104.80 **
Method x Mental Ability	4	212.77	53.19	2.77 *
Model	8	4265.19	533.15	2.77 **
Error	397	7630.58	19.22	
Total	405	11895.77		

** Significant at .01 level
* Significant at .05 level

TABLE 96

ANALYSIS OF VARIANCE OF UNIT 3 RETENTION DATA WHEN SUBJECTS WERE GROUPED BY TEACHER AND PRIOR GENERAL MATHEMATICAL ACHIEVEMENT LEVEL

Source of Variation	df	Sum of Squares	Mean Square	F
Teacher	6	431.57	71.93	4.06 **
Achievement Level	2	3843.79	1921.89	108.50 **
Teacher x Achievement Level	12	547.17	45.60	2.57 **
Model	20	5074.15	253.71	14.32 **
Error	385	6821.62	17.72	
Total	405	11895.77		

** Significant at .01 level

4

TABLE 97

ANALYSIS OF VARIANCE OF UNIT 3 RETENTION DATA WHEN SUBJECTS WERE GROUPED BY TEACHER AND MENTAL ABILITY LEVEL

Source of Variation	df	Sum of Squares	Mean Square	F
Teacher	6	156.59	26.10	1.40
Mental Ability	2	3440.51	1720.25	92.04 **
Teacher x Mental Ability	12	576.19	48.02	2.57 **
Model	20	4699.89	234.99	12.57 **
Error	385	7195.88	18.69	
Total	405	11895.77		

** Significant at .01 level

TABLE 98

ANALYSIS OF TEACHER ATTITUDE DATA

Teacher	X_{pre}	X_{post}	D	D^2
1	8.2	7.4	-0.8	0.64
2	7.8	7.1	-0.7	0.49
3	8.4	8.6	0.2	0.04
4	8.9	8.4	-0.5	0.25
5	9.1	9.1	0.0	0.00
6	9.0	9.0	0.0	0.00
7	8.8	8.8	0.0	0.00
	60.2	58.4	-1.8	1.42

$$N = 7, \; N - 1 = 6$$

$$\Sigma D = -1.8, \; \Sigma D^2 = 1.42$$

$$t = \frac{\Sigma D}{\sqrt{\frac{N\Sigma D^2 - (\Sigma D)^2}{N-1}}} = \frac{-1.8}{\sqrt{\frac{7(1.42-(-1.8)^2}{6}}} = \frac{-1.8}{\sqrt{\frac{6.70}{6}}}$$

$$= \frac{-1.8}{\sqrt{1.11}} = \frac{-1.8}{1.05} = -1.71$$

Thus, $t = -1.71$

TABLE 99

ANALYSIS OF STUDENT ATTUTUDE DATA

Group	N	Mean Gain Score	t	$t_{.95}$	Conclusion
8th Grade - Method D	85	0.01	0.5	1.67	N.S.*
8th Grade - Method E	67	-0.29	-0.6	1.67	N.S.
8th Grade - Method S	72	0.05	0.3	1.67	N.S.
9th Grade - Method D	18	0.12	0.6	1.74	N.S.
9th Grade - Method E	23	0.20	0.5	1.72	N.S.
9th Grade - Method S	29	0.36	1.2	1.70	N.S.

* N.S. - Non-significant at .05 level

APPENDIX D

FIGURES

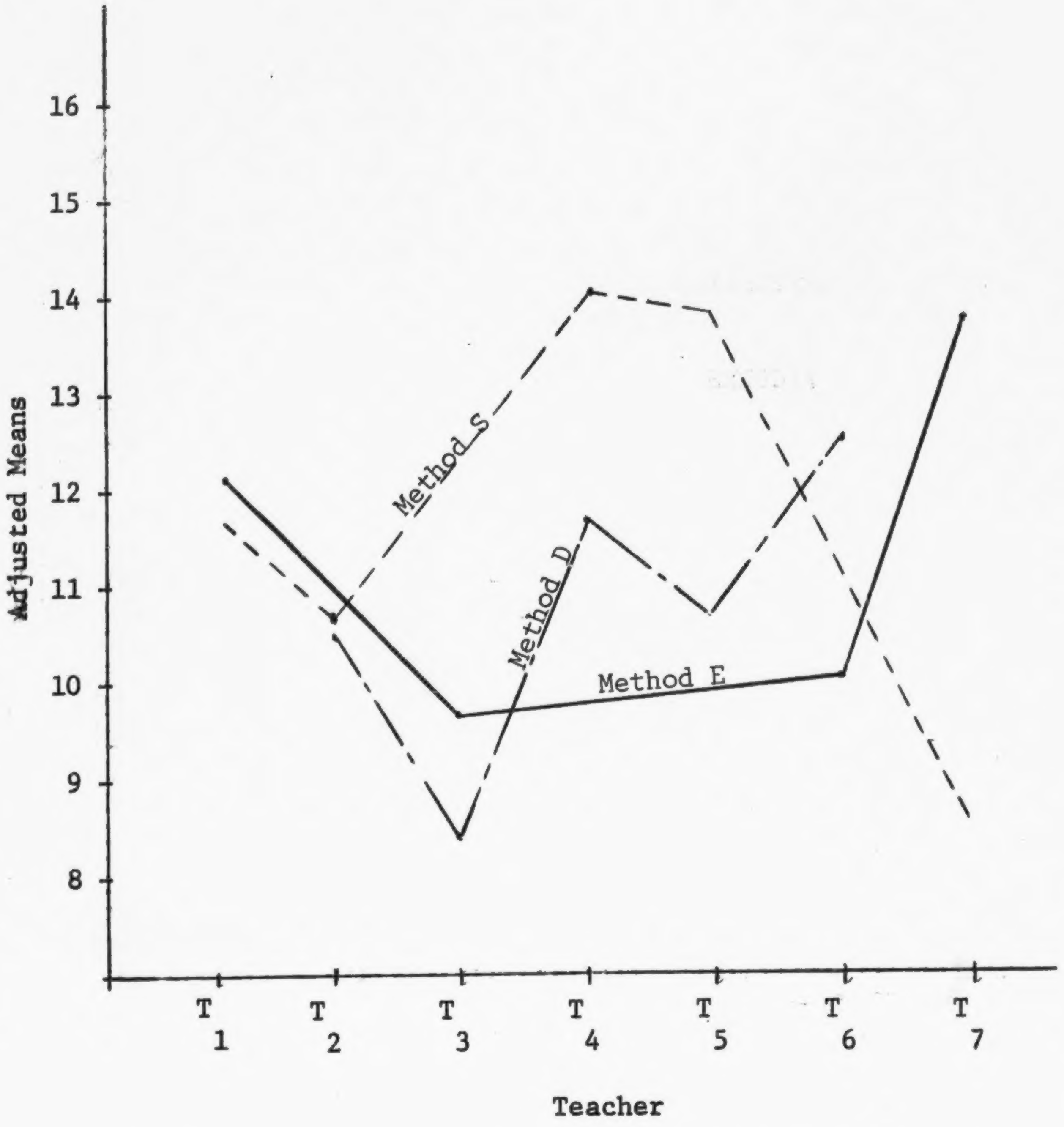


Figure 1. - - Interaction of Method with Teacher on Achievement in Unit 1.

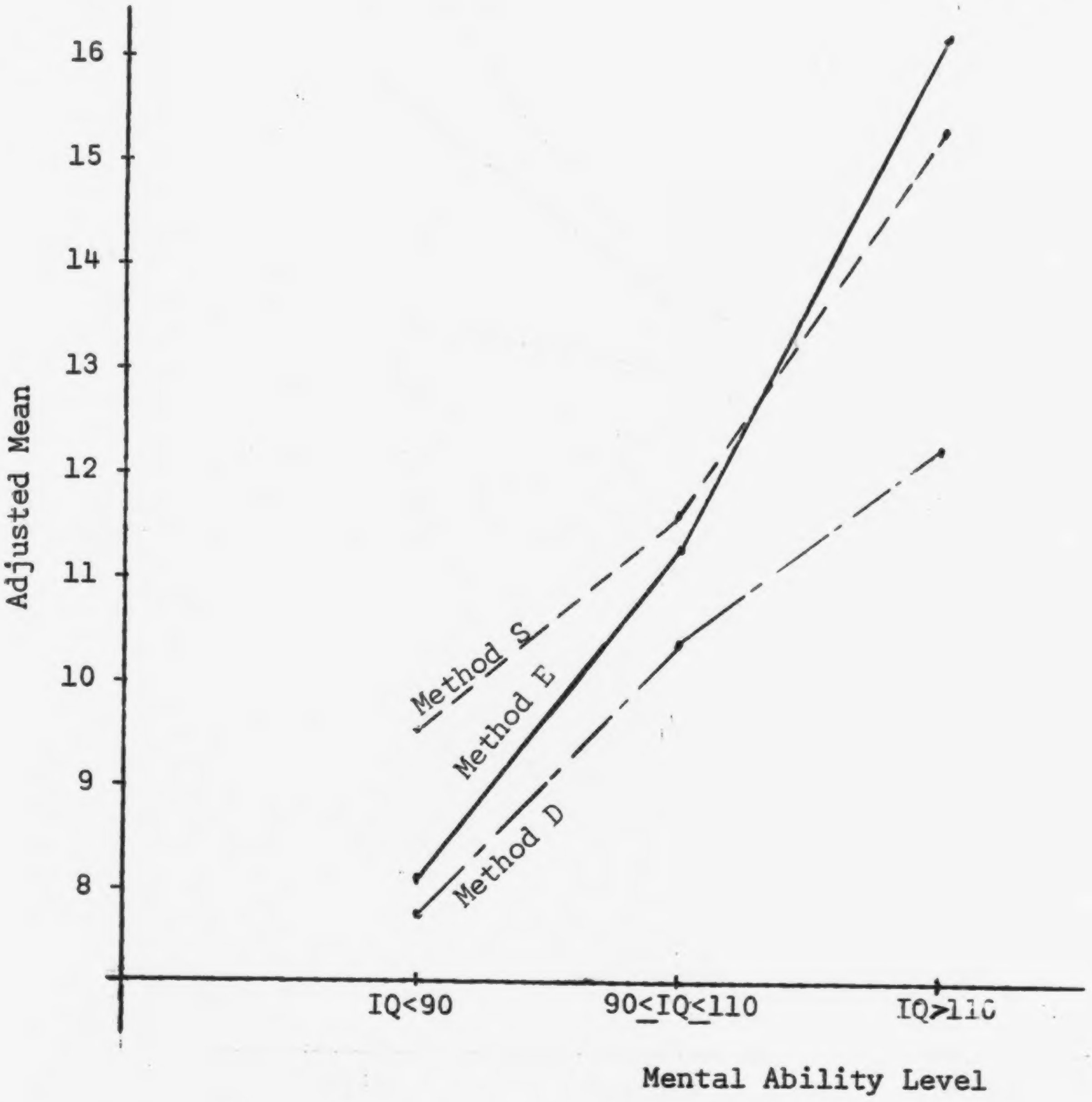


Figure 2.--Interaction of Method with Mental Ability Level on Achievement in Unit 1.

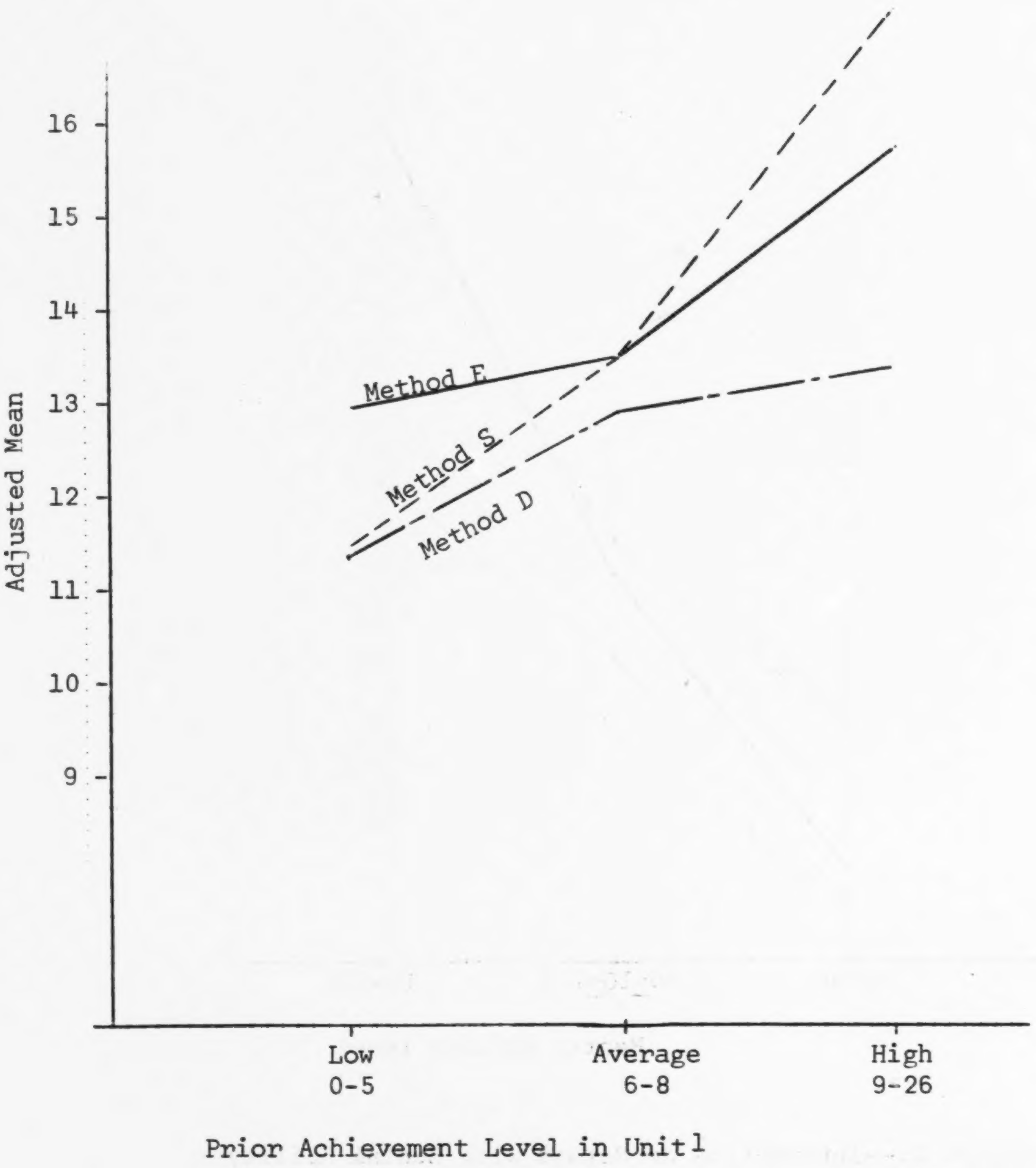


Figure 3.-- Interaction of Method with Prior Achievement Level in Unit 1 When the Criterion was Achievement in Unit 1.

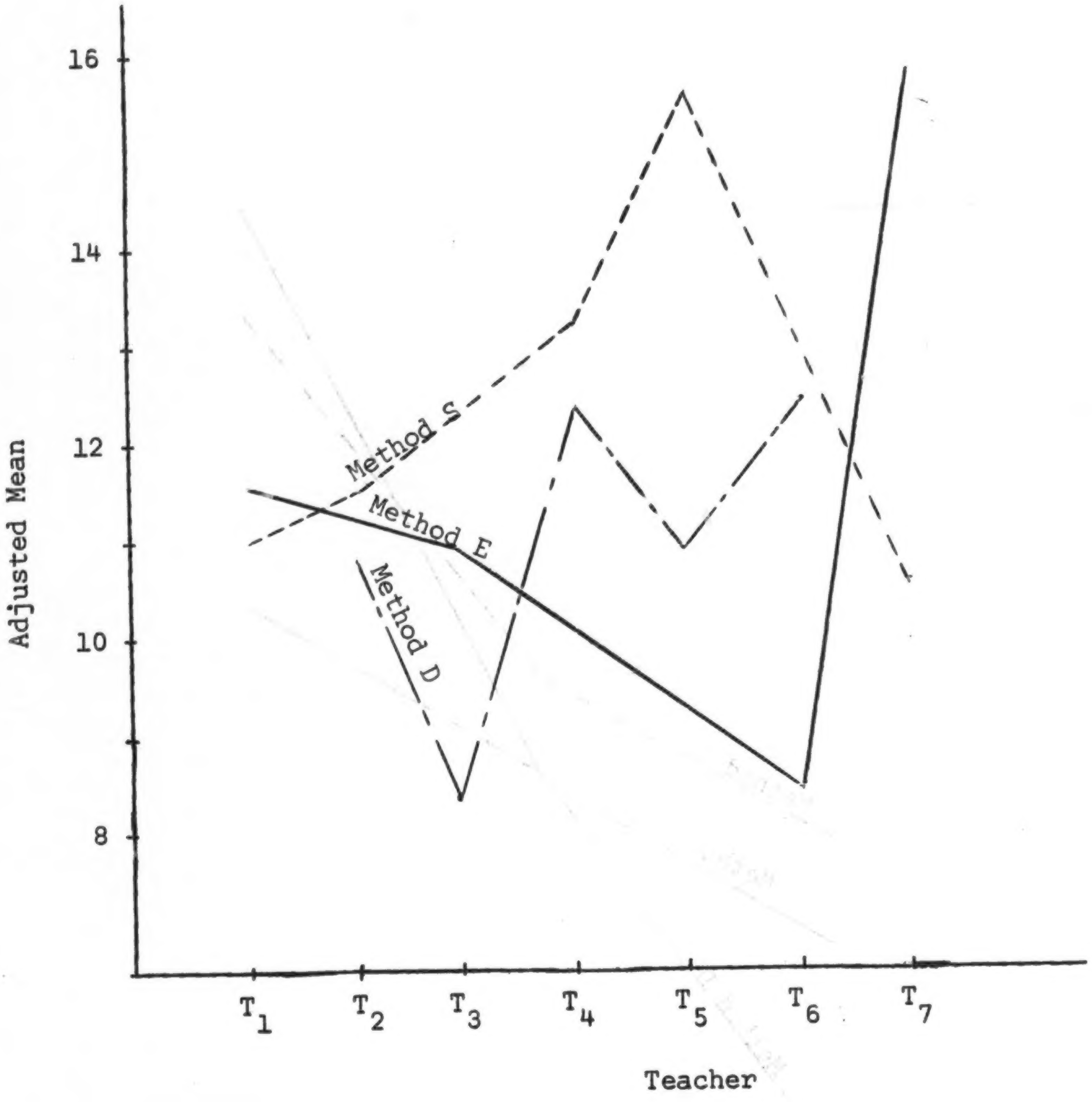


Figure 4.--Interaction of Teacher with Method on Retention in Unit 1.

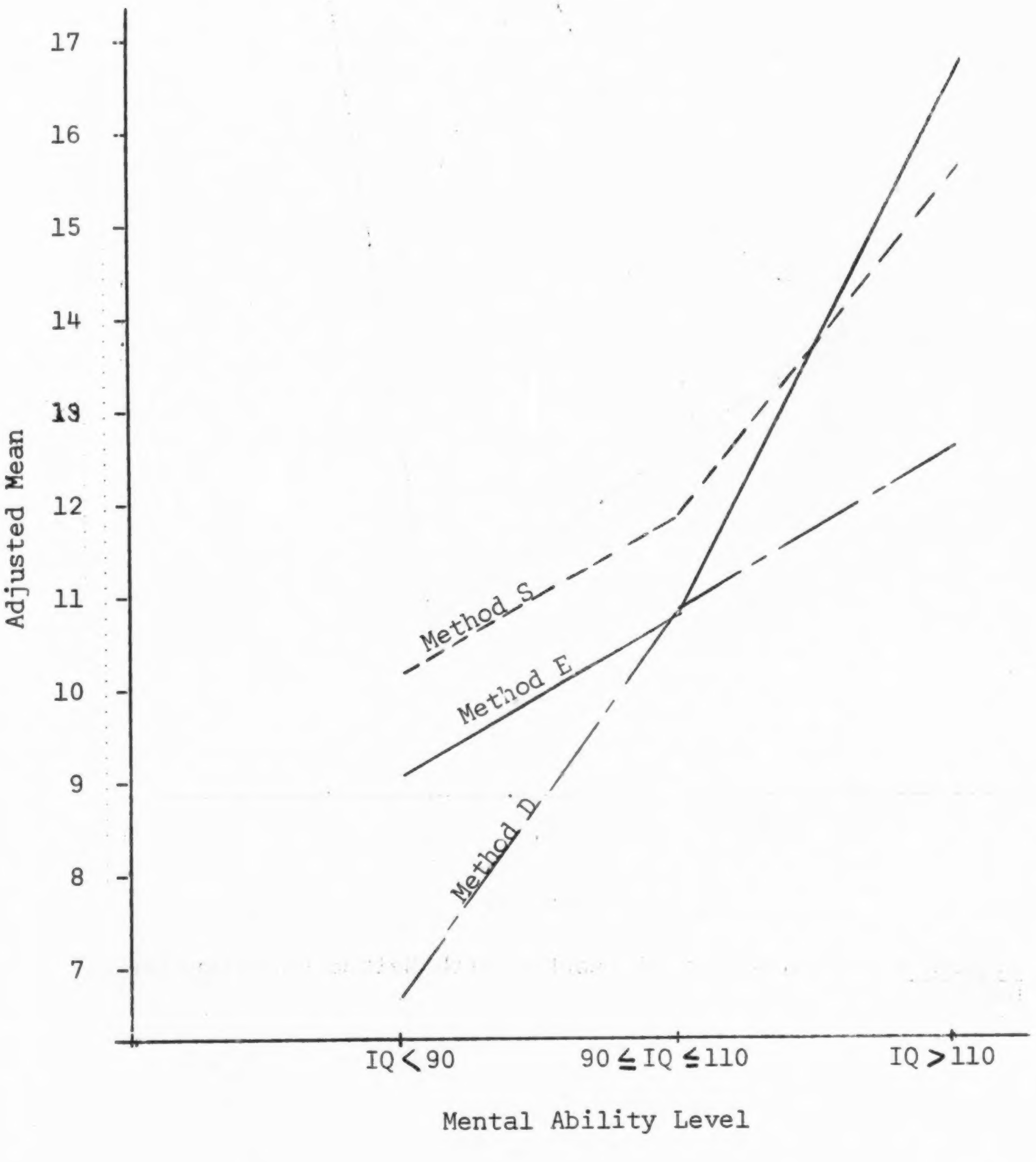


Figure 5. - - Interation of Method with Mental Ability Level on Retention in Unit 1.

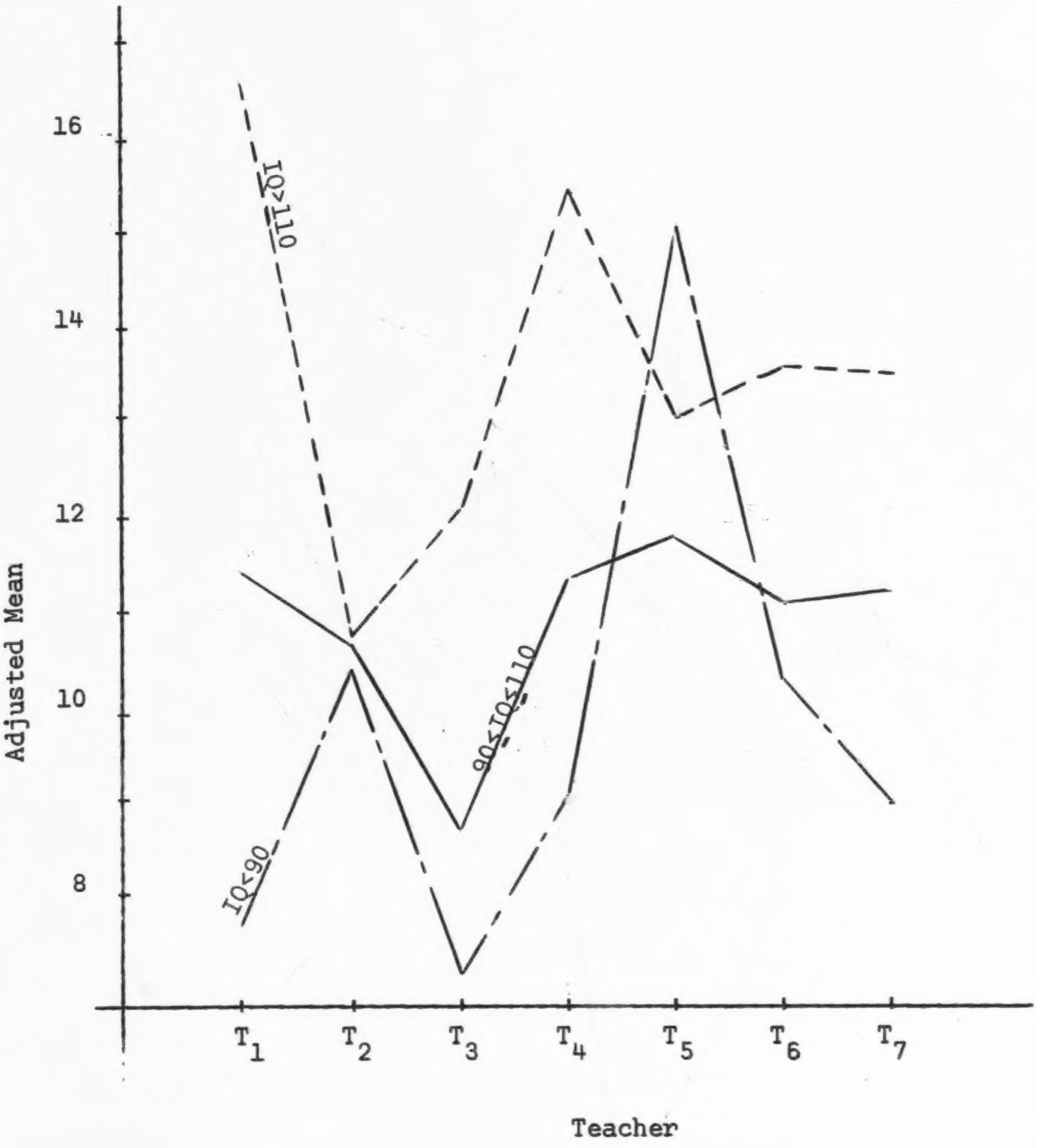


Figure 6.--Interaction of Teacher with Mental Ability Level on Achievement in Unit 1.

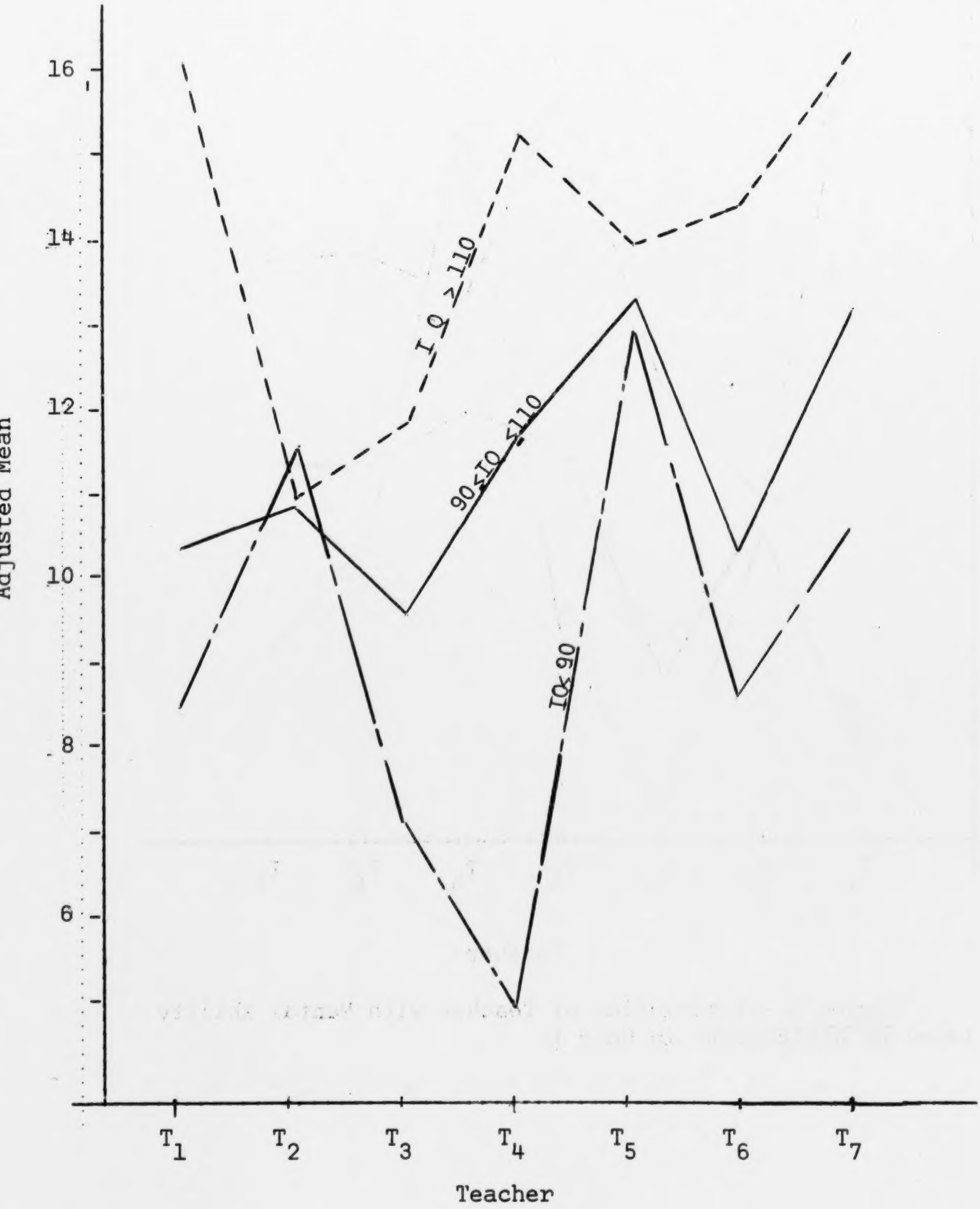


Figure 7.-- Interaction of Teacher with Mental Ability Level on Retention in Unit 1.

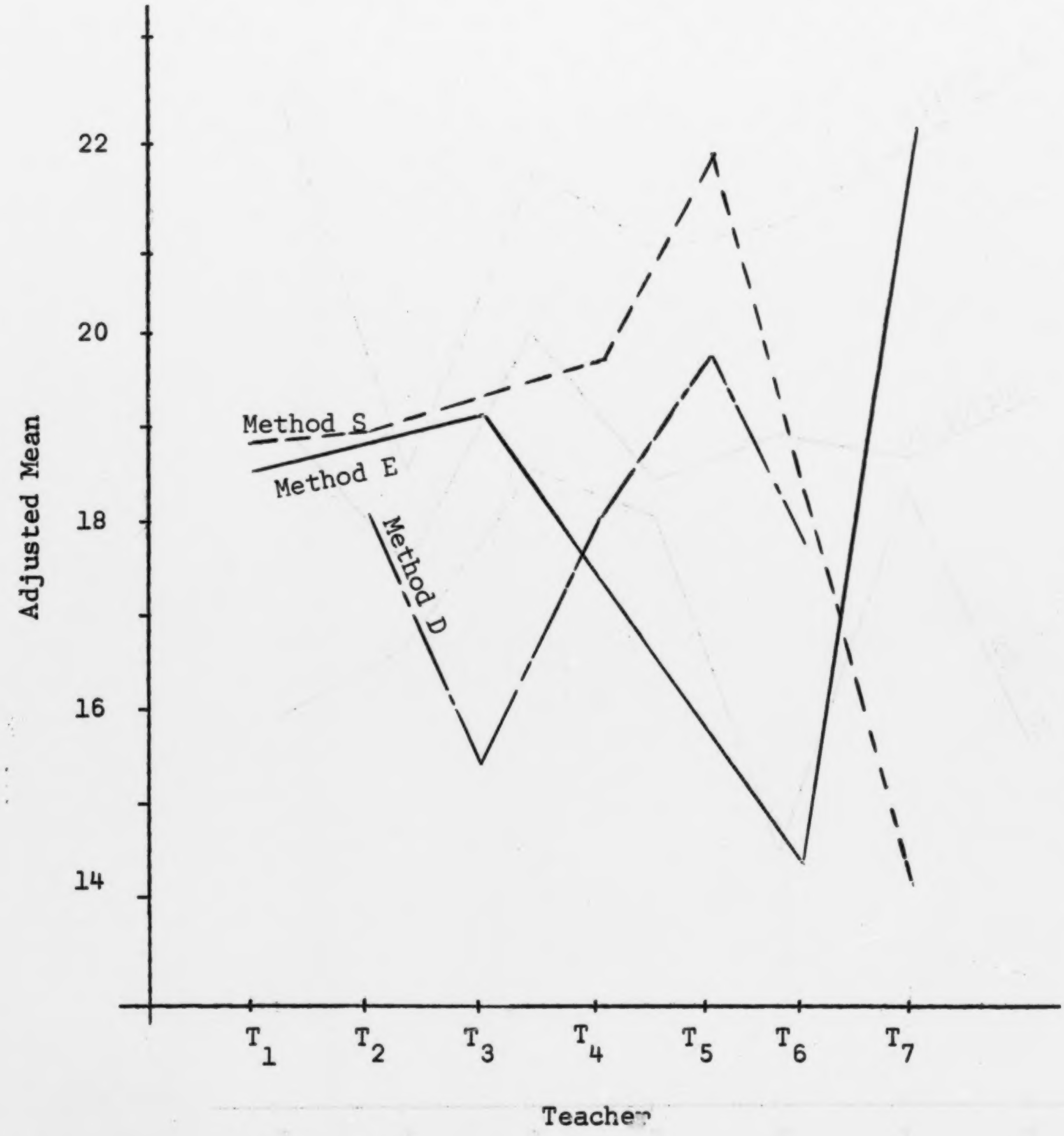


Figure 8.--Interaction of Method with Teacher on Achievement in Unit 3.

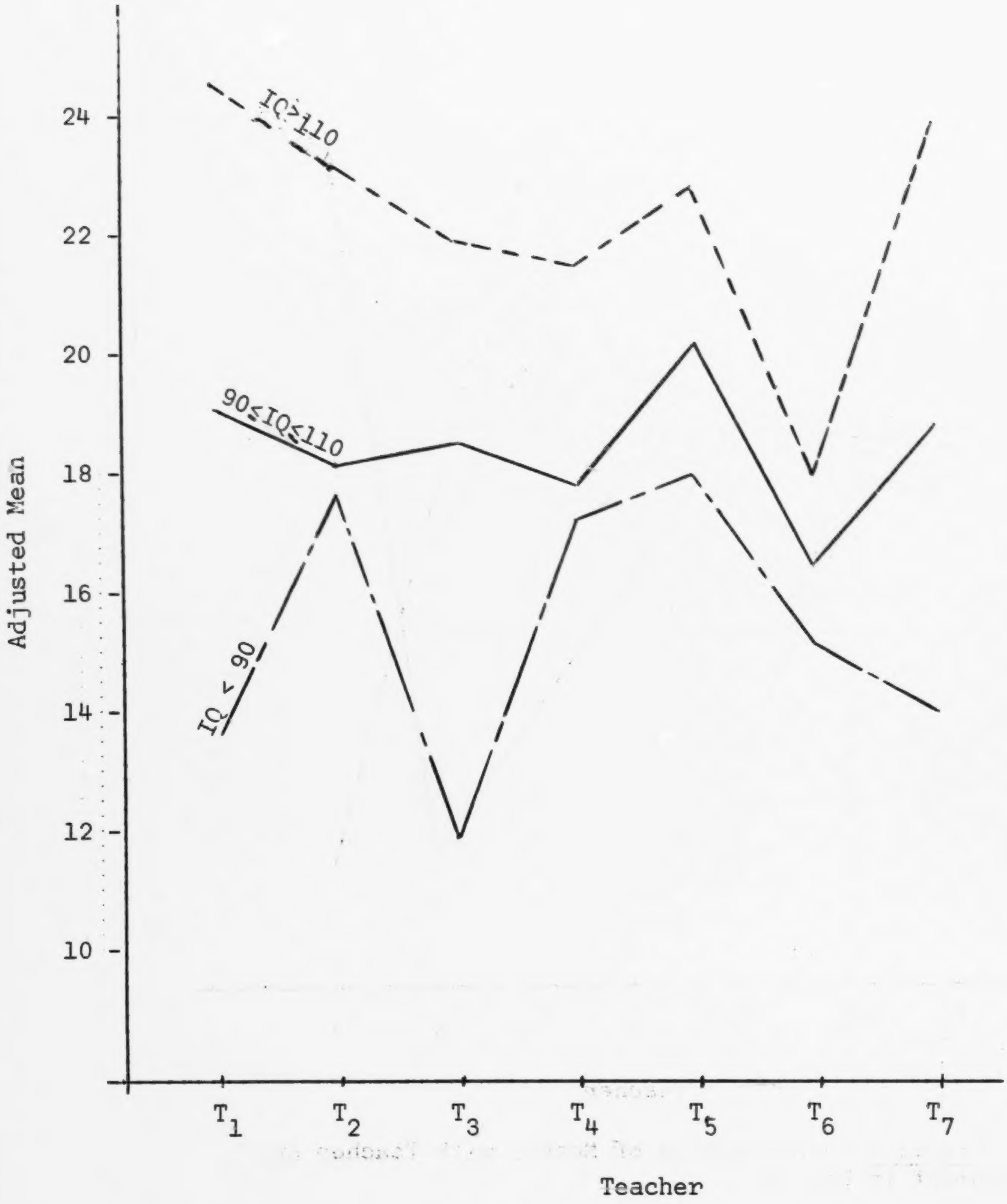


Figure 9.-- Interaction of Teacher with Mental Ability Level on Achievement in Unit 3.

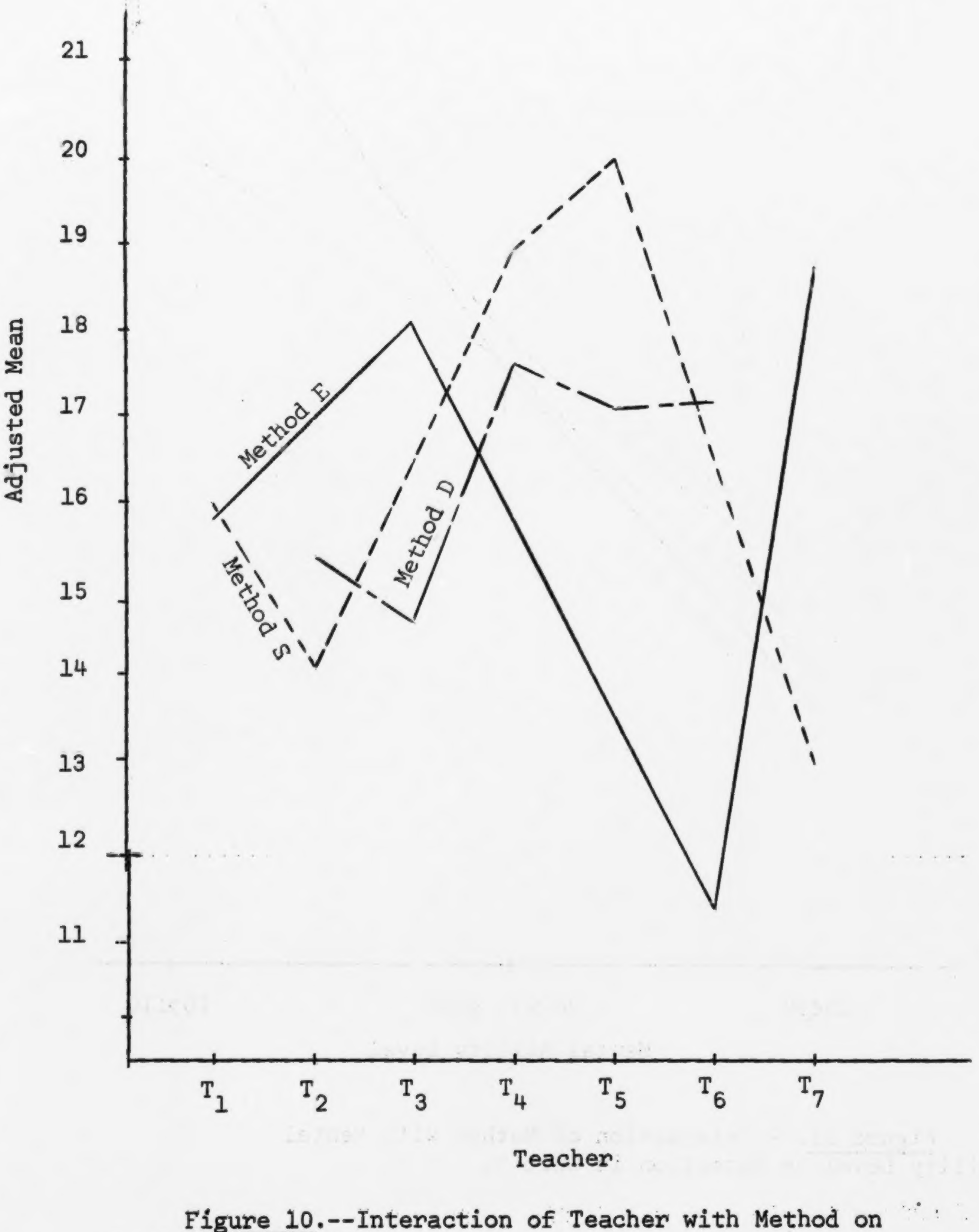


Figure 10.--Interaction of Teacher with Method on Retention in Unit 3.

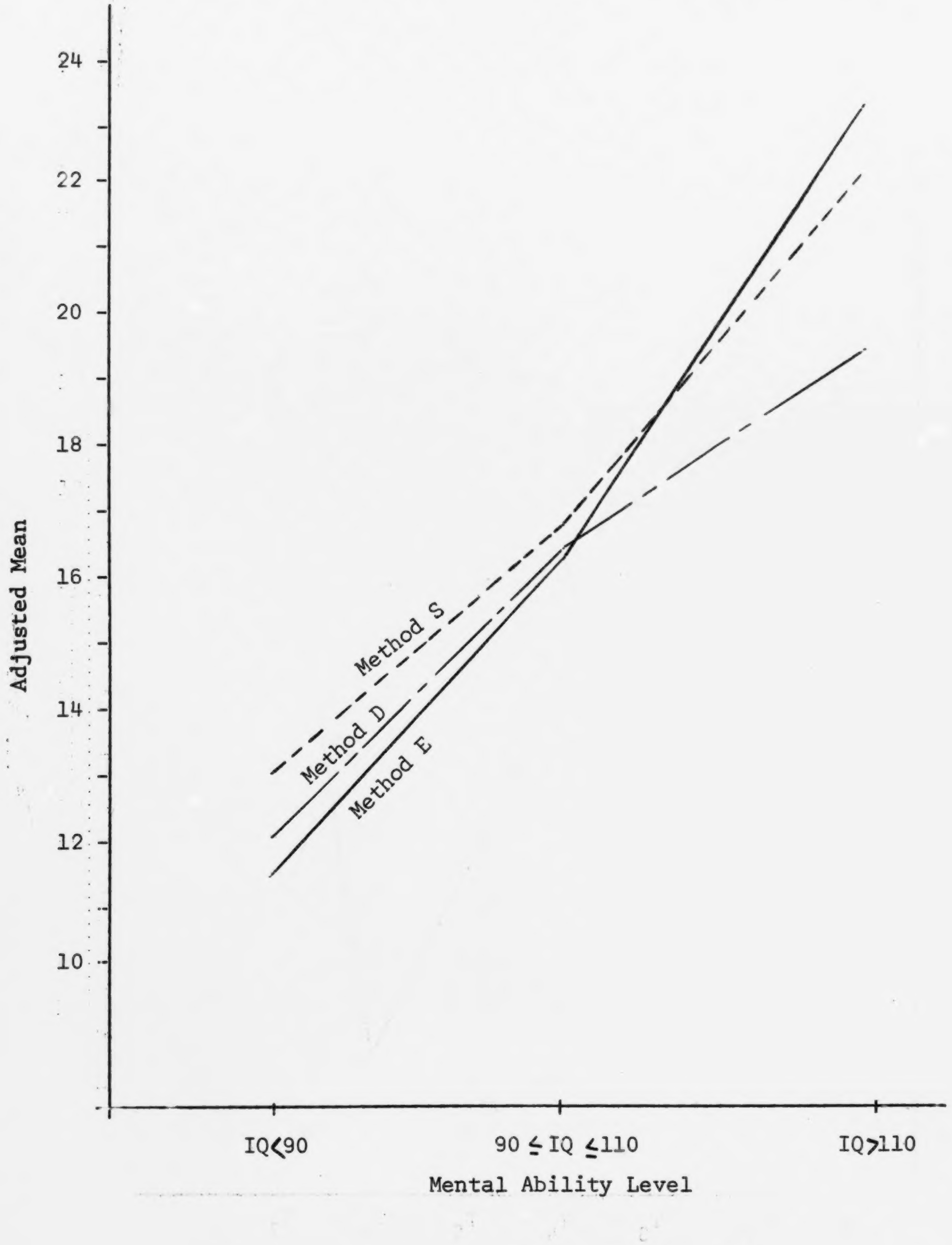


Figure 11.-- Interaction of Method with Mental Ability Level on Retention in Unit 3.

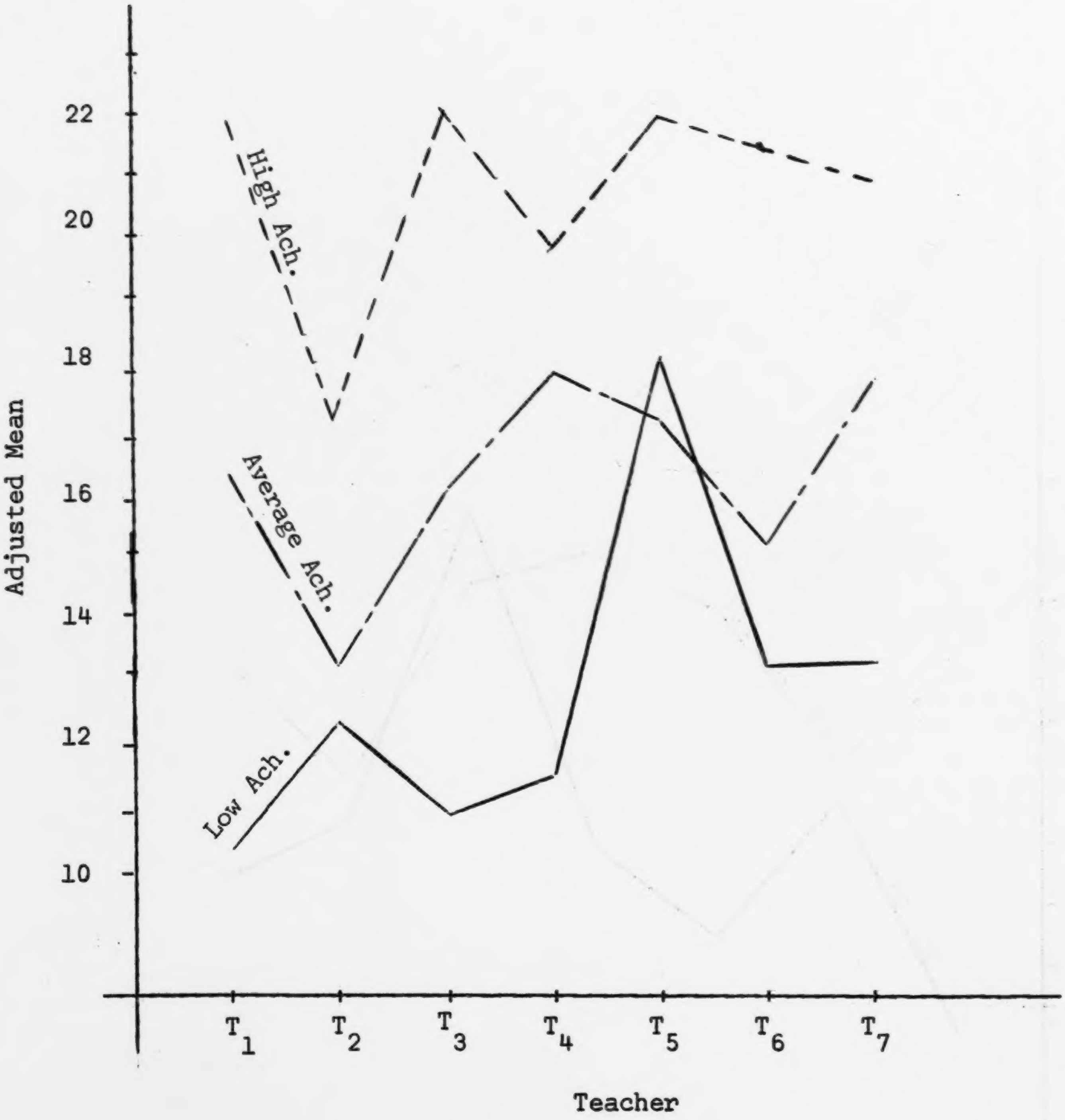


Figure 12.--Interaction of Teacher with Prior General Mathematical Achievement Level on Retention in Unit 3.

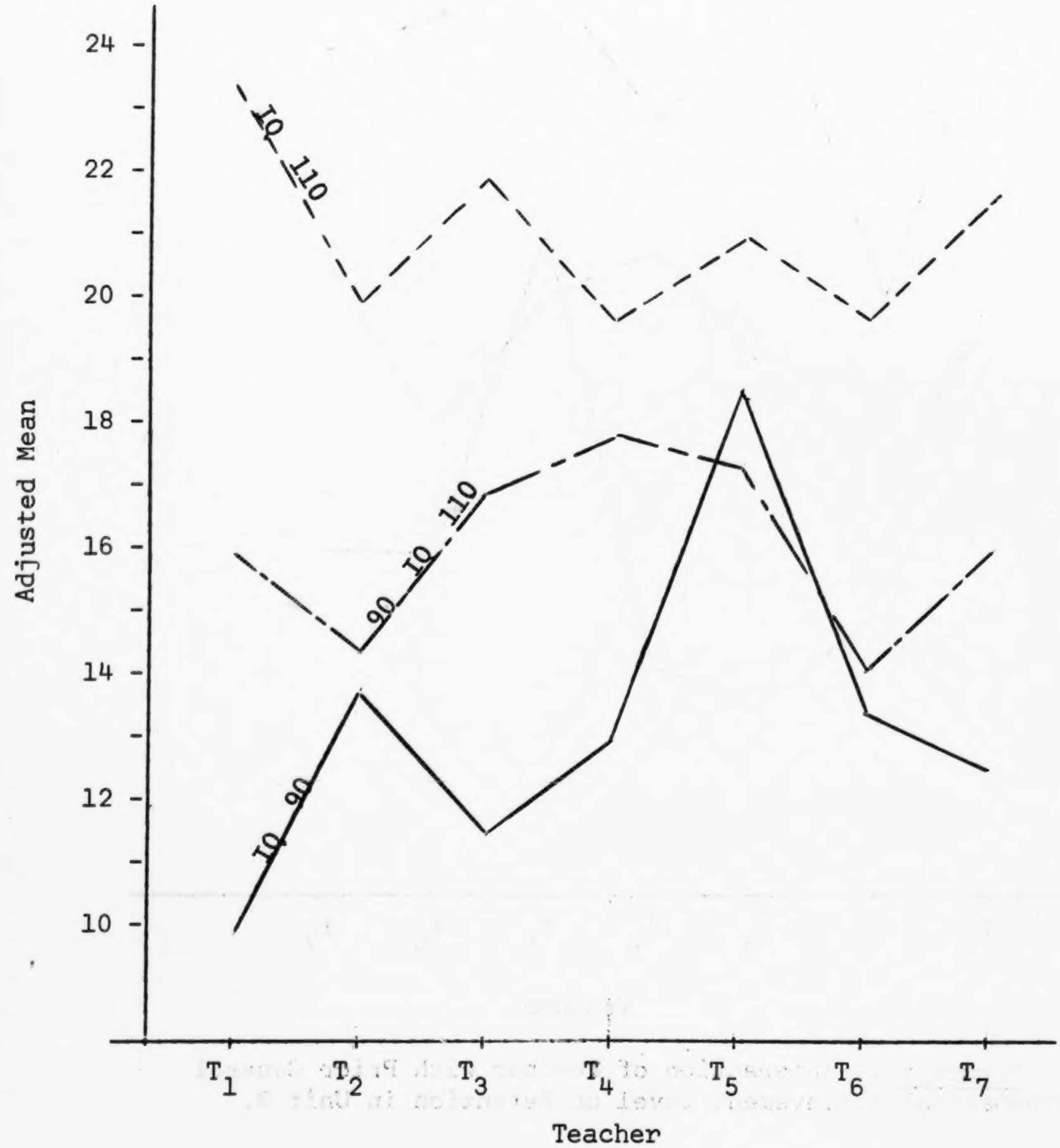


Figure 13.-- Interaction of Teacher with Mental Ability Level on Retention in Unit 3.